Patrik

Outline

I. Reinsurance Loss Reserving Problems

- ◇ **Problem 1: Claim report lags to reinsurers are generally longer, especially for casualty excess losses**
  - The claim report lag (the time from date of accident until first report to the reinsurer) is exacerbated by the lengthy reporting pipeline:
    - ◇ Claim is reported to cedant
    - ◇ Claim filters through cedant's report system to its reinsurance department
    - ◇ Claim travels through an intermediary before finding its way to the reinsurer
    - ◇ Claim appears in reinsurer's claim system
  - Serious claims tend to be under-reserved (i.e. modal reserving practices). This extends the reporting timeline since it takes a while for the claims to exceed the reinsurance threshold
  - Mass tort claims (i.e. asbestosis-related injuries) may have extreme delays in discovery or in reporting to the cedant
- ◇ **Problem 2: There is a persistent upward development of most claim reserves**
  - Caused by three things:
    - ◇ Economic and social inflation
    - ◇ Tendency of claims adjusters to reserve at modal values
    - ◇ Tendency to under-reserve ALAE
- ◇ **Problem 3: Claims reporting patterns differ greatly by reinsurance line, by type of contract, by specific contract terms, by cedant and possibly by intermediary**
  - Exposures assumed by reinsurers tend to be heterogeneous
  - This makes reserving difficult since traditional reserving methods require large volumes of homogeneous data
  - Even when reinsurers have large amounts of similar exposure, low frequency and lengthy report lags may cause extreme fluctuation in historical loss data

◇ **Problem 4: Because of the heterogeneity stated in Problem 3, industry statistics are not very useful**

- Every two years, the Reinsurance Association of America (RAA) publishes a summary of casualty excess reinsurance loss development statistics
- The heterogeneity of the exposure and reporting differences by company must be considered when using these statistics
- No two reinsurers have comparable Schedule P's because the Annual Statement does not properly categorize reinsurance exposures into homogeneous groups
- Most reinsurers' loss reserves are aggregated into one LOB (excess casualty)
- ISO loss development statistics by line are not applicable to reinsurance reserving without significant adjustments to the data. These adjustments may increase the indicated growth
    - ◇ For excess coverage, the lag in reserving or reporting claims grows with the attachment point
    - ◇ Primary company direct statistics do not reflect the additional delays noted in Problem 1

◇ **Problem 5: The reports the reinsurer receives may be lacking important information**

- Most proportional covers require only summary claims information
- Often data are reported by calendar or underwriting year instead of by accident year
- Even when there is individual claims reporting, information tends to be insufficient, requiring reinsurers to pursue more info from the cedant
- It's desirable to have a professional reinsurance claims staff (even when cedant is handling claims) to advise cedant's staff and possibly reduce ultimate payments
- In loss reserving, it's useful to have an exposure measure to compare loss estimates against (such as reinsurance premium by primary LOB)
    - ◇ On most contracts, losses are coded correctly by primary line
    - ◇ Reinsurance premium is assigned to LOB according to the percentage breakdown estimate made at the beginning of the contract. If percentages do not properly reflect loss exposure by primary LOB, comparisons between premiums and losses might be invalid

- For most treaties, there tends to be an added IBNR exposure for both premiums and losses

◇ **Problem 6: Because of the heterogeneity in coverage and reporting requirements, reinsurers often have data coding and IT systems problems**

- Business grows faster than the ability of reinsurers' data systems to handle and produce reports requested by marketing, underwriting, claims, accounting and actuarial staffs

◇ **Problem 7: The size of an adequate loss reserve compared to surplus is greater for a reinsurer**

- More of a management problem (rather than technical)
- Due to the issues described in Problems 1-6, many managers refuse to believe the magnitude of loss liabilities coming from the actuary (especially when IBNR has such a long tail)

◇ **U.S. Tax Reform Act of 1986**

- Requires the discounting of loss reserves for income tax purposes
- Now that insurers must discount loss reserves, they no longer have an implicit risk margin built into their loss reserve estimates
- This buffer flows into profits and is taxed sooner, decreasing assets and increasing companies' risk level

II. Components of a Reinsurer's Loss Reserve

◇ **Component 1: Case reserves reported by the ceding companies**

- Reported on an individual claim basis (excess contracts) or in bulk summary form (proportional contracts)

◇ **Component 2: Reinsurer additional reserves on individual claims**

- Reinsurer reviews individual claims and specifies additional case reserves (ACR) if necessary

◇ **Component 3: Actuarial estimate of future development on components 1 and 2**

- Known as IBNER (Incurred but not enough reserved)

◇ **Component 4: Actuarial estimate of pure IBNR**

- Usually combined with component 3 due to limitations in data systems. Together, components 3 and 4 are known as IBNR

◇ **Component 5: Discount for future investment income**

- Companies can take credit for future investment income on assets supporting certain types of claims, such as WC permanent total cases, auto PIP annuity claims and medical professional liability claims

◇ **Component 6: Risk load**

- Adverse deviation loading is used to keep reserves at a conservative level
- Some actuaries load this in implicitly through conservative assumptions, while others account for it explicitly
- This component is more important for reinsurers due to the long-tailed nature of their exposure

III. A General Procedure

◇ **Step 1: Partition the reinsurance portfolio into reasonably homogeneous exposure groups that are relatively consistent over time with respect to mix of business**

- Segregate contracts and loss exposure into categories of business (listed below in priority order) on the basis of loss development potential
  - ◇ LOB: property, casualty, etc.
  - ◇ Type of contract: facultative, treaty, finite
  - ◇ Type of reinsurance cover: quote share, surplus share, excess per-risk, excess per-occurrence, aggregate excess, cat, etc.
  - ◇ Primary LOB - for casualty
  - ◇ Attachment point - for casualty
  - ◇ Contract terms: flat-rated, retro-rated, sunset clause, share of loss adjustment expense, claims-made vs. occurrence coverage, etc.
  - ◇ Type of cedant: small, large or E&S (excess & surplus) company
  - ◇ Intermediary
- Not necessary to partition data into all eight categories mentioned above (credibility issue)

- Other notes on data partitioning:
  - ◇ Within each category above, the exposure should be further refined by contract type (treaty vs. facultative) and retention type (per-occurrence excess vs. aggregate excess)
  - ◇ Unique claim types (asbestos, pollution, etc.) should be separate
  - ◇ Treaty casualty excess exposure should be segregated by attachment point range and by primary LOB (since they have different report lags)
  - ◇ Treaty casualty proportional exposure should be similarly segregated (is the treaty share of ground-up exposure or share of excess?)
  - ◇ Facultative casualty exposure should be split between primary programs (ground-up exposure) and nonprimary programs (excess exposure)
- It's important to rely on the knowledge of underwriters and other staff members when determining the proper data partition

◇ **Step 2: Analyze the historical development patterns. If possible, consider individual case reserve development and the emergence of IBNR claims separately**

◇ **Step 3: Estimate the future development. If possible, estimate the bulk reserves for IBNER and pure IBNR separately**

- Due to the extreme variability in year-to-year reinsurance data, claim development patterns should be studied over long time periods, as long as the *expected* patterns are reasonably stable from year-to-year
- Since claim development studies are time-consuming, it's best to perform the analysis in the third or fourth quarter of the year (to construct models before the year-end time crunch)
- Once models are created, they can be applied to year-end and quarter-end claims and exposures to estimate IBNR

IV. Claim Report and Payment Lags

◇ When analyzing reinsurance development patterns, it's useful to consider the inverse of the usual chain-ladder development factors. These are known as lags

◇ Lags can then be used to create a graph with time in years on the x-axis and the lag percentage on the y-axis

◇ This graph resembles a probability cdf and can be interpreted as the probability that any particular claims dollar will be reported to the reinsurer by time t

◇ Statistics can be calculated from this curve (such as expected value and standard deviation) to compare one claim pattern to another

V. Methods for Short-Tailed Exposure Categories

◇ Any exposure for which losses are reported and settled quickly

◇ Consists of:

- Treaty property proportional
- Treaty property catastrophe
- Treaty property excess
- Facultative property

◇ Method 1: Set IBNR equal to some percentage of the latest-year EP

◇ Method 2: Reserve up to a selected loss ratio (especially for new LOBs), where the selected loss ratio is larger than the one computed from reported non-cat claims

◇ If losses are summarized by underwriting year, then percentage estimates should be used to allocate losses to true accident years to avoid overstating accident year loss development

VI. Methods for Medium-Tailed Exposure Categories

◇ Any exposure for which claims are almost completely settled within five years and with average aggregate claims dollar report lag of one to two years

◇ Consists of:

- Treaty property excess higher layers (takes a long time to penetrate the layer)
- Construction risks (discovery period can be long)
- Surety (salvage recoveries tend to have long tail)

◇ Method 1: Standard chain-ladder (CL) method

- Advantage is that it strongly correlates future development with an overall lag pattern and with the claims reported for each accident year
- Disadvantage is that IBNR is so correlated with reported claims that estimates are not very credible for recent, immature years
- Possible to use paid losses rather than reported losses to increase stability, but that could backfire for immature years where very few losses have been paid

VII. Methods for Long-Tailed Exposure Categories

◇ Any exposure for which the average aggregate claims dollar report lag is over two years and whose claims are not settled for many years

◇ Consists of:

- Treaty casualty excess (longest lags excluding asbestos, pollution, etc.)
- Treaty casualty proportional
- Asbestos, pollution, etc.

◇ First step is to separate these exposures into more homogeneous groups based on guidance from marketing, underwriting, claims and accounting personnel

◇ **Asbestos, pollution, other health hazard and other mass tort**

- Must be analyzed separately
- No claims for long time periods followed by gigantic claims
- Cannot use traditional reserving methods
- Must rely on complex reserving models

◇ Method 1: Standard chain-ladder method (not great for immature years)

◇ Method 2: Bornhuetter/Ferguson (BF) method

- Advantage is that it correlates future development for each year with an exposure measure (the reinsurance premium multiplied by a selected loss ratio)
- Disadvantages with the BF IBNR estimate include that it is dependent upon the selected loss ratio and that the estimate ignores reported claims for each accident year

◇ **Method 3: Stanard-Bühlmann (Cape Cod) method**

- **Key innovation of the SB method is that the ultimate expected loss ratio for all years combined is estimated from the overall reported claims experience, instead of being selected judgmentally, as in the BF method**
- A **disadvantage** of the SB method is that the IBNR by year is dependent upon rate-level adjusted premium
- Before moving on to the formulas, let's define a few terms:
  - ◇ $SBELR$ = SB estimate of the ELR
  - ◇ $SBIBNR(k)$ = SB IBNR estimate, year k
  - ◇ $RRL(k)$ = reported reinsurance loss, year k

- ◇ $ARPP(k)$ = adjusted risk pure premium, year k
- ◇ $Rlag(k)$ = aggregate claim dollar report lag, year k

- Using the terms above:

$$\boxed{\begin{aligned} SBELR &= \frac{\sum RRL(k)}{\sum[ARPP(k) \cdot Rlag(k)]} \\ SBIBNR(k) &= SBELR \cdot ARPP(k) \cdot (1 - Rlag(k)) \end{aligned}}$$

- The term $ARPP(k) \cdot Rlag(k)$ is also known as the "used-up premium" for year k
- Let's look at an example. Given the following as of December 31, 2014:

AY	Earned Risk Pure Premium	Adjusted Premium	Aggregated Reported Loss	Aggregate Loss Report Lag
2012	1000	1200	800	0.70
2013	1500	1800	500	0.40
2014	2000	2000	400	0.30

- First, calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2012	1200	0.70	$840 = 1200(0.70)$
2013	1800	0.40	720
2014	2000	0.30	600

- Next, we calculate the SB ELR as follows:
  - ◇ SB ELR = reported losses / used-up premium $= (400 + 500 + 800)/(600 + 720 + 840) = 0.787$
- Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
2012	1200	0.70	$283.32 = 1200(0.787)(1-0.70)$
2013	1800	0.40	849.96
2014	2000	0.30	1101.80
Total			**2235.08**

◇ It's also worth mentioning that the SB ultimate losses for an AY are found by adding the SB IBNR to the reported losses to date. Then, we can divide the SB ultimate loss by the **earned risk pure premium** to calculate the final SB ultimate loss ratio

◇ **Method 4: Simple credibility IBNR estimate**

- When we don't have complete confidence in rate-level premium adjustments, we can weight the CL and SB methods together using a credibility factor
- Gives more weight to the SB estimate for immature years
- Gives more weight to the CL estimate for older years where the cumulative rate-level adjustments are less reliable
- Before moving on the formula, let's define a few terms:
  - ◇ Cred$IBNR(k)$ = credibility IBNR for year k
  - ◇ $CLIBNR(k) = CL$ IBNR for year k
  - ◇ CF = credibility factor (between 0 and 1)
  - ◇ $Z(k) = CF \cdot Rlag(k)$
- Using the terms above:

$$\boxed{\text{Cred}IBNR(k) = Z(k) \cdot CLIBNR(k) + (1 - Z(k)) \cdot SBIBNR(k)}$$

- Given a credibility factor of 0.6, let's revisit our example from earlier:

AY	Reported Losses	Aggregate Loss Report Lag	SB IBNR	CL IBNR	Cred IBNR
2012	800	0.70	283.32	342.86	308.33
2013	500	0.40	849.96	750.00	825.97
2014	400	0.30	1101.80	933.33	1071.48
Total			2235.08	2026.19	**2205.78**

- Here are the calculations for AY 2012:
  - ◇ CL IBNR = $342.86 = \frac{800}{0.70} - 800$
  - ◇ Cred IBNR = $308.33 = 0.60(0.70)(342.86) + (1 - 0.60(0.70))(283.32)$

◇ Method 5: Other credibility procedures

- Weight together IBNR estimates based on reported claims and paid claims
  - ◇ Reported claims include case reserves that vary over time depending on the claims adjuster setting the reserve
  - ◇ Paid claims tend to be more stable, assuming you have sufficient data and believe the data to have a consistent expected payment pattern
  - ◇ Weights could be based on relative claim report and payment lags for each year

- Use the ELR inherent in the underlying pricing of the exposure in lieu of or in conjunction with the SB ELR
  - ◇ We call these *a priori* ELR estimates and can use them as our BF ELR estimates. These can then be used to calculate BF IBNR
  - ◇ Weight this *a priori* IBNR against the CL IBNR (Benktander Method)

◇ Method 6: Alternative estimation methodologies

- Stochastic reserving models
  - ◇ An advantage is that they give us more information and provide insight not available with traditional methods
  - ◇ A disadvantage is that they tend to be complicated and black boxish, making them difficult to interpret and explain to management
- Claim count/claim severity model
  - ◇ An advantage is that we can build models for various lag distributions, and then connect them with models for the dollar reserving and the payments on individual claims

VIII. Monitoring and Testing Predictions

◇ Monitoring and testing quarterly claims run-off against predictions provides early warning of problems

◇ For short and medium-tailed lines, past AY run-off can be compared with previous year-end reported open reserves and IBNR reserves

◇ More sophistication required for long-tailed lines

◇ If more claims emerge than what was expected, what does it mean?

- Is it purely random?
- Does it indicate that the beginning IBNR was too small (or lags too short)? If our lags are too short (i.e. our reporting pattern is not long enough), then our beginning IBNR estimate will be too small (assuming we are running a chain-ladder method or any other method that relies on a reporting pattern to calculate IBNR)

◇ Continue to monitor claims each quarter to see if pattern persists. If so, we may need to lengthen the lags

IX. Final Comments

◇ Differences between primary company loss reserving and reinsurance reserving:

- Less information
- Longer report and settlement timing delays
- Low frequency and high severity claims

Original Mathematical Problems & Solutions

MP #1

Given the following as of December 31, 2012:

AY	Earned Risk Pure Premium	Adjusted Premium	Aggregated Reported Loss	Aggregate Loss Report Lag
2010	$500	$800	375	0.70
2011	800	900	250	0.40
2012	1,000	1,000	200	0.30

Estimate the total IBNR using the Stanard-Bühlmann method.

Solution:

◇ First, calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2010	800	0.70	$560 = 800(0.70)$
2011	900	0.40	360
2012	1000	0.30	300

◇ Next, calculate the SB ELR:

- SB ELR = reported losses / used-up premium = $(375+250+200)/(560+360+300) = 0.676$

◇ Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
2010	800	0.70	$162.24 = 800(0.676)(1-0.70)$
2011	900	0.40	365.04
2012	1000	0.30	473.20
Total			$\boxed{\$1{,}000.48}$

MP #2

Given the following:

AY	Adjusted Premium	Claims at 12/31/12	Claim Report Lag at 12/31/12	Claim Report Lag at 6/30/13	Actual Claims at 6/30/13
2008	$2,700	$2,500	0.95	0.98	$2,500
2009	3,000	2,100	0.85	0.90	2,310
2010	4,200	1,500	0.70	0.77	1,800
2011	4,000	1,100	0.50	0.58	1,265
2012	6,000	2,400	0.30	0.41	3,060

a) Using the chain-ladder method, calculate the difference between the actual claims and the expected claims at 6/30/13. Give two possible reasons for the difference.

b) Using the credibility method with a credibility factor of 0.4, calculate the difference between the actual claims and the expected claims at 6/30/13. Give two possible reasons for the difference.

Solution to part a:

◇ First, calculate the CL IBNR as of 12/31/12:

AY	CL IBNR
2008	$131.579 = (2500/.95) - 2500$
2009	370.588
2010	642.857
2011	1100
2012	5600

◇ Next, calculate the expected claims at 6/30/13:

AY	Expected Claims at 6/30/13
2008	$2578.947 = 2500 + 131.579\left(\frac{0.98-0.95}{1-0.95}\right)$
2009	2223.529
2010	1650
2011	1276
2012	3280

◇ Calculate the difference between the actual claims and the expected claims at 6/30/13:

AY	Expected Claims at 6/30/13	Actual Claims at 6/30/13	Difference
2008	2578.947	2500	$-78.947 = 2500 - 2578.947$
2009	2223.529	2310	86.471
2010	1650	1800	150
2011	1276	1265	-11
2012	3280	3060	-220
Total			$\boxed{-\$73.48}$

◇ **Two possible reasons for the difference are that the beginning IBNR was too large OR pure randomness**

Solution to part b:

◇ First, calculate the SB ELR as of 12/31/12:

AY	Used-Up Premium	Claims at 12/31/12
2008	$2565 = 2700(0.95)$	2500
2009	2550	2100
2010	2940	1500
2011	2000	1100
2012	1800	2400
Total	11855	9600

Thus, the SB ELR $= 9600/11855 = 0.810$

◇ Next, calculate the SB IBNR as of 12/31/12:

AY	SB IBNR
2008	$109.35 = 0.810(2700)(1 - 0.95)$
2009	364.50
2010	1020.60
2011	1620
2012	3402

◇ Next, calculate the Credibility IBNR as of 12/31/12:

AY	CF	$Z(k)$	Credibility IBNR
2008	0.4	$0.38 = 0.4(0.95)$	$117.797 = 131.579(0.38) + 109.35(1 - 0.38)$
2009	0.4	0.34	366.570
2010	0.4	0.28	914.832
2011	0.4	0.20	1516
2012	0.4	0.12	3665.76

◇ Next, calculate the expected claims at 6/30/13:

AY	Expected Claims at 6/30/13
2008	$2570.678 = 2500 + 117.797\left(\frac{0.98-0.95}{1-0.95}\right)$
2009	2222.19
2010	1713.461
2011	1342.56
2012	2976.048

◇ Calculate the difference between the actual claims and the expected claims at 6/30/13:

AY	Expected Claims at 6/30/13	Actual Claims at 6/30/13	Difference
2008	2570.678	2500	$-70.678 = 2500 - 2570.678$
2009	2222.19	2310	87.81
2010	1713.461	1800	86.539
2011	1342.56	1265	-77.56
2012	2976.048	3060	83.952
Total			$110.06

◇ **Two possible reasons for the difference are that the beginning IBNR was too small OR pure randomness**

Original Essay Problems

EP #1

Briefly describe the six components of a reinsurer's loss reserve.

EP #2

Identify the three steps involved in reinsurer loss reserving.

EP #3

a) Define claim report lag $R(t)$ in terms of the standard chain-ladder age-to-ultimate development factor.

b) Explain how the claim report lag can be interpreted as a probability cumulative distribution function. Give one reason why this interpretation is useful.

EP #4

Given the following exposure types:

- ◇ Short-tailed exposures
- ◇ Medium-tailed exposures
- ◇ Long-tailed exposures

a) Identify one loss reserve estimation method for each exposure type above.

b) Provide two examples of each exposure type above.

EP #5

Patrik describes a credibility procedure that weights the chain-ladder IBNR with the Stanard-Bühlmann IBNR.

a) Briefly explain the rationale behind this procedure.

b) Describe two alternative credibility procedures.

Original Essay Solutions

ES #1

- ◇ Component 1: Case reserves reported by the ceding companies
  - Reported on an individual claim basis (excess contracts) or in bulk summary form (proportional contracts)
- ◇ Component 2: Reinsurer additional reserves on individual claims
  - Reinsurer reviews individual claims and specifies additional case reserves if necessary
- ◇ Component 3: Actuarial estimate of future development on components 1 and 2
  - Known as IBNER (incurred but not enough reserved)
- ◇ Component 4: Actuarial estimate of pure IBNR
  - Usually combined with component 3 due to limitations in data systems. Together, components 3 and 4 are known as IBNR
- ◇ Component 5: Discount for future investment income
  - Companies can take credit for future investment income on assets supporting certain types of claims, such as WC permanent total cases, auto PIP annuity claims and medical professional liability claims
- ◇ Component 6: Risk load
  - Adverse deviation loading is used to keep reserves at a conservative level

ES #2

- ◇ Partition the reinsurance portfolio into reasonably homogeneous exposure groups that are relatively consistent over time with respect to mix of business
- ◇ Analyze the historical development patterns. If possible, consider individual case reserve development and the emergence of IBNR claims separately
- ◇ Estimate the future development. If possible, estimate the bulk reserves for IBNER and pure IBNR separately

ES #3

Part a:

- ◇ The claim report lag at time t is the inverse of the chain-ladder age-to-ultimate development factor

Part b:

- ◇ The claim report lag can be read as the probability that any particular claims dollar will be reported to the reinsurer by time t. This view allows us to compute statistics of the claims reporting process, enabling us to compare one claim report pattern with another

ES #4

Part a:

- ◇ Short-tailed – set IBNR equal to some percentage of the latest-year earned premium
- ◇ Medium-tailed – chain-ladder method
- ◇ Long-tailed – Cape Cod method

Part b:

- ◇ Short-tailed – treaty property proportional, treaty property catastrophe
- ◇ Medium-tailed – treaty property excess higher layers, construction risk
- ◇ Long-tailed – treaty casualty excess, asbestos

ES #5

Part a:

- ◇ Due to the difficulty in obtaining rate-level adjusted premium, we may not have complete confidence in the Stanard-Bühlmann approach. Thus, we can weight the chain-ladder and Stanard-Bühlmann methods together using a credibility factor

Part b:

- ◇ Weight together IBNR estimates based on reported claims and paid claims. The weights could be based on relative claim report and payment lags for each year

- ◇ Use the ELR inherent in the underlying pricing of the exposure in lieu of or in conjunction with the Stanard-Bühlmann ELR. This ELR can be used to calculate BF IBNR. We then weight this BF IBNR with the chain-ladder IBNR

Past CAS Exam Problems & Solutions

2019 #13

Given the following information for a reinsurer as of December 31, 2018:

Accident Year	Earned Risk Pure Premium ($000)	Adjusted Premium ($000)	Aggregate Reported Loss ($000)	Aggregate Loss Report Lag
2015	6,800	7,400	4,500	90%
2016	7,200	7,700	3,400	70%
2017	7,600	7,900	2,300	50%
2018	8,000	8,000	3,000	30%

- ◇ Report lag reaches 100% in the 5^{th} year

a) Calculate the IBNR estimate for accident year 2018 as of December 31, 2018 using the Stanard-Bühlmann (Cape Cod) method.

b) Calculate the IBNR estimate for accident year 2018 as of December 31, 2018 using the chain-ladder method.

c) Calculate the IBNR estimate for accident year 2018 as of December 31, 2018 using a credibilty-weighted combination of the Stanard-Bühlmann and chain-ladder methods given a selected credibility factor of 0.6

d) Describe a situation in which an actuary would choose to use a credibility-weighted combination of the Stanard-Bühlmann and chain-ladder methods.

Solution to part a:

- ◇ The SB ELR is $\frac{4500+3400+2300+3000}{7400(0.90)+7700(0.70)+7900(0.50)+8000(0.30)} = 0.717$
- ◇ The 2018 SB IBNR $= 8000(0.717)(1-0.30) = \boxed{\$4{,}015{,}200}$

Solution to part b:

- ◇ The 2018 CL IBNR $= \frac{3000}{0.30} - 3000 = \boxed{\$7{,}000{,}000}$

Solution to part c:

- ◇ $Z = 0.60(0.30) = 0.18$
- ◇ The 2018 Cred. IBNR $= (0.18)(7000) + (1-0.18)(4015.2) = \boxed{\$4{,}552{,}464}$

Solution to part d:

- ◇ When we don't have complete confidence in rate-level premium adjustments, we may not want to solely use the SB method. Additionally, we may not have complete confidence in the chain-ladder method due to immature data and overly-leveraged LDFs. If both of these occur at the same time, we may want to use a credibility weighted average of the two methods

2019 #14

Identify and briefly describe four major problems that make reinsurance loss reserving more difficult than primary insurer loss reserving.

Solution:

- ◇ Data lags to reinsurers are longer than those for primary insurers. Claims are reported to the primary insurer first. They are not reported to the reinsurer until a certain threshold is reached
- ◇ There is persistent upward development of losses for reinsurers. This is often caused by ALAE under-reserving by the primary insurer as well as primary insurer case reserves being set at the mode
- ◇ Due to heterogeneity of coverages, industry statistics are not very useful
- ◇ Due to heterogeneity of coverages, there are often IT and/or data issues for reinsurers

2018 #14

a) A claims person, faced with insufficient and possibly conflicting information about a potentially serious claim, may tend to reserve to "expectation," which is most likely interpreted by the claims person as the mode of the probability distribution.

Briefly describe two ways this practice can make reinsurance reserving difficult.

b) Briefly describe three reasons heterogeneous exposures assumed by a reinsurance company can make reinsurance reserving difficult.

Solution to part a:

- ◇ Losses and expenses will be consistently under-reserved for serious claims
- ◇ The mode is likely to be low due to the majority of claims being small. This increases the report lag to the reinsurer since it will take more time for large claims to breach the reinsurance threshold

Solution to part b:

- ◇ Industry statistics re not very useful
- ◇ Difficult to aggregate data to increase credibility
- ◇ Data coding and IT issues due to the many different requirements of heterogeneous data

2018 #16

Given the following information:

Accident Year	Rate-Level Adjusted Premium ($000)	Aggregate Reported Loss at Dec. 31, 2016 ($000)	Aggregate Loss Report Lag Dec. 31, 2016	Aggregate Reported Loss at Jun. 30, 2017 ($000)	Aggregate Loss Report Lag at Jun. 30, 2017
2014	6,000	3,000	0.70	3,500	0.75
2015	7,000	2,000	0.50	2,200	0.55
2016	10,000	4,000	0.30	4,800	0.35
Total	23,000	9,000	N/A	10,500	N/A

- ◇ The selected credibility factor is 0.50

a) Test the total expected claims emergence from December 31, 2016 to June 30, 2017 against the actual claims emergence using a credibility-weighted combination of the Stanard-Bühlmann and chain-ladder methods.

b) Provide two reasons which would explain the difference calculated in part a. above.

Solution to part a:

- ◇ SB ELR $= \frac{3000+2000+4000}{6000(0.70)+7000(0.50)+10000(0.30)} = 0.841$
- ◇ AY 2014
  - CL IBNR at 12/31/16 $= 3000/0.7 - 3000 = 1285.71$
  - SB IBNR at 12/31/16 $= 6000(0.841)(1 - 0.70) = 1513.80$
  - $Z = 0.70(0.50) = 0.35$
  - Cred IBNR at 12/31/16 $= 1285.71(0.35) + 1513.80(1 - 0.35) = 1433.97$
  - Expected emergence at 06/30/17 $= 1433.97\left(\frac{0.75-0.70}{1-0.70}\right) = 239.00$
- ◇ AY 2015
  - CL IBNR at 12/31/16 $= 2000/0.5 - 2000 = 2000.00$
  - SB IBNR at 12/31/16 $= 7000(0.841)(1 - 0.50) = 2943.50$
  - $Z = 0.50(0.50) = 0.25$
  - Cred IBNR at 12/31/16 $= 2000.00(0.25) + 2943.50(1 - 0.25) = 2707.63$
  - Expected emergence at 06/30/17 $= 2707.63\left(\frac{0.55-0.50}{1-0.50}\right) = 270.76$
- ◇ AY 2016
  - CL IBNR at 12/31/16 $= 4000/0.3 - 4000 = 9333.33$
  - SB IBNR at 12/31/16 $= 10000(0.841)(1 - 0.30) = 5887.00$
  - $Z = 0.30(0.50) = 0.15$
  - Cred IBNR at 12/31/16 $= 9333.33(0.15) + 5887.00(1 - 0.15) = 6403.95$
  - Expected emergence at 06/30/17 $= 6403.95\left(\frac{0.35-0.30}{1-0.30}\right) = 457.43$
- ◇ Total expected emergence $239.00 + 270.76 + 457.43 = 967.18$ and actual emergence is 10500 - 9000 = 1500.
- ◇ Thus, the total expected emergence is 532.82 less than the actual emergence

Solution to part b:

- ◇ The initial IBNR was too low
- ◇ Pure randomness

2017 #14

Briefly describe six reasons why reinsurers have larger uncertainty in their loss reserves than primary insurers.

Solution:

- ◇ Data lags to reinsurers are longer than those for primary insurers
- ◇ There is persistent upward development of losses for reinsurers
- ◇ Due to heterogeneity of coverages, industry statistics are not very useful
- ◇ Due to heterogeneity of coverages, there are often IT and/or data issues for reinsurers
- ◇ Reporting patterns can vary by many dimensions like cedant, intermediary, type of contract, attachment point, etc.
- ◇ Reports available to reinsurers may lack important information as they are often at a summarized level

2017 #15

Briefly describe how the following components of a loss reserve could vary for a reinsurer versus a primary insurer.

a) Additional case reserves on individual claims

b) Pure IBNR

c) Risk Load

Solution to part a:

- ◇ Reinsurers might add additional reserves on individual claims based on their own expertise. This is different from a primary insurer who has no need for additional reserves since they set the original reserves

Solution to part b:

- ◇ Due to system limitations for reinsurers, they often combine IBNR and IBNER into a single IBNR number. Primary insurers do not typically have these data limitations

Solution to part c:

- ◇ This component is often more important for reinsurers due to the long-tailed nature of their exposure

2017 #16

Given the following information ($000) as of December 31, 2016:

Accident Year	Earned Risk Pure Premium	Rate-Level Adjusted Premium	Aggregate Reported Loss	Aggregate Loss Report Lag	Chain Ladder IBNR	Bornhuetter-Ferguson IBNR
2012	5,000	5,000	3,500	0.91	350	364
2013	4,500	5,500	3,300	0.80	825	900
2014	5,000	6,000	2,750	0.67	1,375	1,667
2015	6,500	6,500	2,500	0.40	3,750	3,300
2016	7,000	7,000	2,000	0.20	8,000	4,800

a) Calculate IBNR for accident year 2016 using the Stanard-Bühlmann method.

b) Briefly describe one advantage and one disadvantage of the Stanard-Bühlmann method.

c) Comment on an appropriate IBNR selection for accident year 2016.

Solution to part a:

- ◇ The SB ELR is calculated as $\frac{3500+3300+2750+2500+2000}{5000(0.91)+5500(0.80)+6000(0.67)+6500(0.40)+7000(0.20)} = 0.828$
- ◇ The SB IBNR for AY 2016 is $7000(0.828)(1-0.20) = \boxed{\$4{,}636.80}$

Solution to part b:

- ◇ Advantage – More stable than the chain-ladder method at early maturities
- ◇ Disadvantage – Requires rate-level adjusted premiums which can be difficult to obtain at times

Solution to part c:

- ◇ The reported losses for AY 2016 appear higher than one might expect based on the older years. Since it's possible that this reflects a change in loss experience, we want to give it some credibility. Thus, I would use a credibility-weighted IBNR estimate between the SB and CL methods since the SB IBNR does not account enough for the large reported loss in AY 2016

2016 #16

Given the following reinsurance company data ($000) as of December 31, 2015:

Calendar-Accident Year	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss	Reported Loss Development Factor to Ultimate
2012	20,500	21,000	9,000	1.250
2013	21,500	27,000	6,000	2.500
2014	22,800	25,000	4,000	5.000
2015	24,000	24,000	3,250	8.000
Total	88,800	97,000	22,250	n/a

- ◇ The selected credibility factor is 0.80

a) Use the Standard-Bühlmann method to estimate the IBNR for all years combined.

b) Use the credibility weighting of the Standard-Bühlmann and chain-ladder methods as presented by Patrik to estimate the IBNR for accident year 2014.

Solution to part a:

◇ First, calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2012	21000	$0.800 = \frac{1}{1.25}$	$16800 = 21000(0.800)$
2013	27000	0.400	10800
2014	25000	0.200	5000
2015	24000	0.125	3000

◇ Next, calculate the SB ELR:

- SB ELR = reported losses / used-up premium = $(9000+6000+4000+3250)/(16800+10800+5000+3000) = 0.625$

◇ Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
2012	21000	0.800	$2625 = 21000(0.625)(1-0.800)$
2013	27000	0.400	10125
2014	25000	0.200	12500
2015	24000	0.125	13125
Total			$\boxed{\$38{,}375{,}000}$

Solution to part b:

◇ The chain-ladder IBNR for AY 2014 is 4000(5-1) = 16000

◇ The SB IBNR for AY 2014 is 12500

◇ The credibility weight is the credibility factor times the report lag. Thus, $Z = 0.80(0.20) = 0.16$

◇ The credibility weighted IBNR is calculated as $0.16(16000)+(1-0.16)(12500) = \boxed{\$13{,}060{,}000}$

2016 #17

Consider the statement:

Primary insurer loss reserving is more straightforward than reinsurance loss reserving because (1) claim report lags to reinsurers are generally longer, and (2) claims reporting patterns differ greatly by reinsurance line and type of contract.

a) Identify two other technical problems with reinsurance reserving that support the above statement.

b) Briefly describe two underlying causes of each technical problem identified in part a. above.

Solution to part a:

- ◇ Persistent upward development of claims
- ◇ Industry reinsurance data may not be useful due to heterogeneity

Solution to part b:

- ◇ Persistent upward development is caused by under-reserving of ALAE, cedants reserving at the mode, and increasing inflation
- ◇ Industry data heterogeneity may be caused by aggregation of cedant LOBs into one LOB for reinsurance reporting. In addition, RAA data is only distributed once every two years

2015 #12

Given the following ground-up information for insurance claims subject to a single excess-of-loss reinsurance treaty as of December 31, 2014:

Claim Number	Occurrence Number	Date of Loss	Reported ALAE	Reported Loss	Total Reported Loss and ALAE
1	1	Feb. 29, 2013	$12,000	$80,000	$92,000
2	2	May 28, 2013	18,000	145,000	163,000
3	3	Sep. 5, 2013	8,000	70,000	78,000
4	3	Sep. 5, 2013	3,250	30,000	33,250
5	4	Jan. 24, 2014	9,000	90,000	99,000
6	5	Mar. 7, 2014	20,000	225,000	245,000
7	6	Oct. 18, 2014	3,750	55,000	58,750

Accident Year	Historical Per-Occurrence Retention
Jan. 1, 2013 – Dec. 31, 2013	$150,000
Jan. 1, 2014 – Dec. 31, 2014	200,000

- ◇ The historical per-occurrence retentions apply to combined loss and ALAE
- ◇ The reinsurance treaty stipulates that the reinsurer must be notified of any occurrences whose reported loss and ALAE reaches 50% of the retention

Demonstrate how the data above indicates the presence of two aspects of reinsurance loss reserving that make it somewhat more difficult than primary loss reserving.

◇ Issue 1: Claim reported lags to reinsurers are generally longer

- The reinsurer would not know about claim 5 because it has not hit 50% of the retention threshold. However, it's likely that it will breach the threshold

◇ Issue 2: Heterogeneity of patterns

- Reinsurers have very heterogeneous exposures. Exposures vary by LOB, contract terms, etc. In this case, the retention changed in 2014. Thus, historical losses before 2014 cannot be directly compared to losses after 2014

2015 #13

Given the following information for a reinsurer as of December 31, 2014 ($000):

Accident Year	Earned Premium	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss	Aggregate Loss Report Lag
2011	10,000	10,000	9,000	6,000	90%
2012	11,500	11,000	12,000	5,000	70%
2013	12,500	12,000	11,000	2,000	40%
2014	14,000	13,000	13,000	4,000	30%

a) Using the Stanard-Bühlmann Method, estimate the IBNR for all accident years combined.

b) Identify and briefly explain the need for the modification made to earned risk pure premium to derive adjusted premium.

Solution to part a:

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2011	9000	0.90	$8100 = 9000(0.90)$
2012	12000	0.70	8400
2013	11000	0.40	4400
2014	13000	0.30	3900

◇ Calculate the SB ELR:

- SB ELR $= \frac{\text{reported losses}}{\text{used-up premium}} = \frac{6000+5000+2000+4000}{8100+8400+4400+3900} = 0.685$

◇ Calculate the SB IBNR for each AY:

- AY 2011 SB IBNR $= 9000(0.685)(1-0.90) = 616.50$
- AY 2012 SB IBNR $= 12000(0.685)(1-0.70) = 2466$
- AY 2013 SB IBNR $= 11000(0.685)(1-0.40) = 4521$
- AY 2014 SB IBNR $= 13000(0.685)(1-0.30) = 6233.50$
- The total SB IBNR is $\boxed{\$13{,}837{,}000}$

Solution to part b:

◇ Current rate level. We want all of the premiums to be on the same level because we combining all years to calculate a single ELR

2014 #13

Describe four aspects of reinsurance loss reserving that make it somewhat more difficult than primary loss reserving.

Solution:

- ◇ Longer development pattern than primary insurer due to:
  - Extended by cedant's reporting pipeline
  - Cedants tend to under-reserve large claims
  - Extreme delays in searching and reporting latent claims
- ◇ Consistent upward development, due to:
  - Inflation impact
  - Cedants tend to under-reserve ALAE
  - Cedants tend to initially reserve large claims at modal values
- ◇ Industry data is not helpful due to:
  - No breakdown of reinsurer's exposures into homogeneous groups
  - Severity of development increases with attachment point
- ◇ Claim development is extremely different due to:
  - Reinsurance contracts are unique
  - Significant fluctuation during development because of single large claims

2014 #14

Given the following reinsurance company data ($000) evaluated as of December 31, 2013:

Calendar/ Accident Year	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss
2010	5,000	5,000	3,000
2011	4,500	5,500	2,500
2012	5,000	6,000	2,500
2013	6,500	6,500	1,500
Total	20,000	23,000	9,500

Reported Loss Development Factors to Ultimate	
12 months	2.50
24 months	2.00
36 months	1.60
48 months	1.25
60 months	1.00

- ◇ Adjusted premium incorporates changes in primary rates, underwriting, and exposure affecting the loss potential

Calculate the Stanard-Bühlmann (Cape Cod) ultimate loss ratio for 2011.

Solution:

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2010	5000	$0.800 = \frac{1}{1.25}$	$4000 = 5000(0.80)$
2011	5500	0.625	3437.5
2012	6000	0.500	3000
2013	6500	0.400	2600

◇ Calculate the SB ELR:

- SB ELR $= \frac{\text{reported losses}}{\text{used-up premium}} = \frac{9500}{4000+3437.5+3000+2600} = 0.729$

◇ Calculate the SB IBNR for AY 2011:

- SB IBNR $= 5500(0.729)(1 - 0.625) = 1503.5625$

◇ Calculate the ultimate loss ratio for AY 2011:

- Ultimate loss = reported loss + SB IBNR $= 2500 + 1503.5625 = 4003.5625$
- Ultimate loss ratio $= \frac{\text{ultimate losses}}{\text{earned premium}} = \frac{4003.5625}{4500} = \boxed{0.89}$

2013 #10

Given the following information ($000) as of December 31, 2012:

Accident Year	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss	Aggregate Loss Report Lag
2010	9,200	9,200	3,600	70%
2011	8,500	7,400	2,500	50%
2012	10,000	10,000	4,200	30%

a) Use the Stanard-Bühlmann method to estimate the IBNR for accident years 2010 through 2012 combined.

b) Use a credibilty-weighted combination of the Stanard-Bühlmann and chain-ladder estimates to calculate a total IBNR estimate for accident years 2010 through 2012 combined. Assume credibility is a linear function of the report lag with a credibility factor of 0.35.

c) Briefly describe one advantage and one disadvantage of the Stanard-Bühlmann method as compared to the chain-ladder method.

Solution to part a:

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2010	9200	0.70	$6440 = 9200(0.70)$
2011	7400	0.50	3700
2012	10000	0.30	3000

◇ Calculate the SB ELR:

- SB ELR $= \frac{\text{reported losses}}{\text{used-up premium}} = \frac{3600+2500+4200}{6440+3700+3000} = 0.784$

◇ Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
2010	9200	0.70	$2163.84 = 9200(0.784)(1-0.70)$
2011	7400	0.50	2900.80
2012	10000	0.30	5488.00
Total			$\boxed{\$10{,}552.64}$

Solution to part b:

AY	CL IBNR	Z	Cred IBNR
2010	$1542.857 = 3600(\frac{1}{0.70}-1)$	$0.245 = 0.35(0.70)$	$2011.699 = 1542.857(0.245) + 2163.840(1-0.245)$
2011	2500	0.175	2830.660
2012	9800	0.105	5940.760
Total			$\boxed{\$10{,}783.12}$

Solution to part c:

◇ Advantage – more stable in most recent accident years

◇ Disadvantage – requires rate-adjusted earned premium which may be difficult to obtain