

# Patrik

## Opening Thoughts

I really like the Cape Cod method. The math is simple and you don't have to think too hard.

## Outline

### I. Reinsurance Loss Reserving Problems

- ◇ **Problem 1: Claim report lags to reinsurers are generally longer, especially for casualty excess losses**
  - The claim report lag (the time from date of accident until first report to the reinsurer) is exacerbated by the lengthy reporting pipeline:
    - ◇ Claim is reported to cedant
    - ◇ Claim filters through cedant's report system to its reinsurance department
    - ◇ Claim travels through an intermediary before finding its way to the reinsurer
    - ◇ Claim appears in reinsurer's claim system
  - Serious claims tend to be under-reserved (i.e. modal reserving practices). This extends the reporting timeline since it takes a while for the claims to exceed the reinsurance threshold
  - Mass tort claims (i.e. asbestosis-related injuries) may have extreme delays in discovery or in reporting to the cedant
- ◇ **Problem 2: There is a persistent upward development of most claim reserves**
  - Caused by three things:
    - ◇ Economic and social inflation
    - ◇ Tendency of claims adjusters to reserve at modal values
    - ◇ Tendency to under-reserve ALAE
- ◇ **Problem 3: Claims reporting patterns differ greatly by reinsurance line, by type of contract, by specific contract terms, by cedant and possibly by intermediary**
  - Exposures assumed by reinsurers tend to be heterogeneous

- This makes reserving difficult since traditional reserving methods require large volumes of homogeneous data
- Even when reinsurers have large amounts of similar exposure, low frequency and lengthy report lags may cause extreme fluctuation in historical loss data

◇ **Problem 4: Because of the heterogeneity stated in Problem 3, industry statistics are not very useful**

- Every two years, the Reinsurance Association of America (RAA) publishes a summary of casualty excess reinsurance loss development statistics
- The heterogeneity of the exposure and reporting differences by company must be considered when using these statistics
- No two reinsurers have comparable Schedule P's because the Annual Statement does not properly categorize reinsurance exposures into homogeneous groups
- Most reinsurers' loss reserves are aggregated into one LOB (excess casualty)
- ISO loss development statistics by line are not applicable to reinsurance reserving without significant adjustments to the data. These adjustments may increase the indicated growth
  - ◇ For excess coverage, the lag in reserving or reporting claims grows with the attachment point
  - ◇ Primary company direct statistics do not reflect the additional delays noted in Problem 1

◇ **Problem 5: The reports the reinsurer receives may be lacking important information**

- Most proportional covers require only summary claims information
- Often data are reported by calendar or underwriting year instead of by accident year
- Even when there is individual claims reporting, information tends to be insufficient, requiring reinsurers to pursue more info from the cedant
- It's desirable to have a professional reinsurance claims staff (even when cedant is handling claims) to advise cedant's staff and possibly reduce ultimate payments
- In loss reserving, it's useful to have an exposure measure to compare loss estimates against (such as reinsurance premium by primary LOB)
  - ◇ On most contracts, losses are coded correctly by primary line

- ◇ Reinsurance premium is assigned to LOB according to the percentage breakdown estimate made at the beginning of the contract. If percentages do not properly reflect loss exposure by primary LOB, comparisons between premiums and losses might be invalid
- For most treaties, there tends to be an added IBNR exposure for both premiums and losses
- ◇ **Problem 6: Because of the heterogeneity in coverage and reporting requirements, reinsurers often have data coding and IT systems problems**
  - Business grows faster than the ability of reinsurers' data systems to handle and produce reports requested by marketing, underwriting, claims, accounting and actuarial staffs
- ◇ **Problem 7: The size of an adequate loss reserve compared to surplus is greater for a reinsurer**
  - More of a management problem (rather than technical)
  - Due to the issues described in Problems 1-6, many managers refuse to believe the magnitude of loss liabilities coming from the actuary (especially when IBNR has such a long tail)
- ◇ **U.S. Tax Reform Act of 1986**
  - Requires the discounting of loss reserves for income tax purposes
  - Now that insurers must discount loss reserves, they no longer have an implicit risk margin built into their loss reserve estimates
  - This buffer flows into profits and is taxed sooner, decreasing assets and increasing companies' risk level

## II. Components of a Reinsurer's Loss Reserve

- ◇ **Component 1: Case reserves reported by the ceding companies**
  - Reported on an individual claim basis (excess contracts) or in bulk summary form (proportional contracts)
- ◇ **Component 2: Reinsurer additional reserves on individual claims**
  - Reinsurer reviews individual claims and specifies additional case reserves (ACR) if necessary

- ◇ **Component 3: Actuarial estimate of future development on components 1 and 2**
  - Known as IBNER (Incurred but not enough reserved)
- ◇ **Component 4: Actuarial estimate of pure IBNR**
  - Usually combined with component 3 due to limitations in data systems. Together, components 3 and 4 are known as IBNR
- ◇ **Component 5: Discount for future investment income**
  - Companies can take credit for future investment income on assets supporting certain types of claims, such as WC permanent total cases, auto PIP annuity claims and medical professional liability claims
- ◇ **Component 6: Risk load**
  - Adverse deviation loading is used to keep reserves at a conservative level
  - Some actuaries load this in implicitly through conservative assumptions, while others account for it explicitly
  - This component is more important for reinsurers due to the long-tailed nature of their exposure

### III. A General Procedure

- ◇ **Step 1: Partition the reinsurance portfolio into reasonably homogeneous exposure groups that are relatively consistent over time with respect to mix of business**
  - Segregate contracts and loss exposure into categories of business (listed below in priority order) on the basis of loss development potential
    - ◇ LOB: property, casualty, etc.
    - ◇ Type of contract: facultative, treaty, finite
    - ◇ Type of reinsurance cover: quote share, surplus share, excess per-risk, excess per-occurrence, aggregate excess, cat, etc.
    - ◇ Primary LOB - for casualty
    - ◇ Attachment point - for casualty
    - ◇ Contract terms: flat-rated, retro-rated, sunset clause, share of loss adjustment expense, claims-made vs. occurrence coverage, etc.

- ◇ Type of cedant: small, large or E&S (excess & surplus) company
- ◇ Intermediary
- Not necessary to partition data into all eight categories mentioned above (credibility issue)
- Other notes on data partitioning:
  - ◇ Within each category above, the exposure should be further refined by contract type (treaty vs. facultative) and retention type (per-occurrence excess vs. aggregate excess)
  - ◇ Unique claim types (asbestos, pollution, etc.) should be separate
  - ◇ Treaty casualty excess exposure should be segregated by attachment point range and by primary LOB (since they have different report lags)
  - ◇ Treaty casualty proportional exposure should be similarly segregated (is the treaty share of ground-up exposure or share of excess?)
  - ◇ Facultative casualty exposure should be split between primary programs (ground-up exposure) and nonprimary programs (excess exposure)
- It's important to rely on the knowledge of underwriters and other staff members when determining the proper data partition
- ◇ **Step 2: Analyze the historical development patterns. If possible, consider individual case reserve development and the emergence of IBNR claims separately**
- ◇ **Step 3: Estimate the future development. If possible, estimate the bulk reserves for IBNER and pure IBNR separately**
  - Due to the extreme variability in year-to-year reinsurance data, claim development patterns should be studied over long time periods, as long as the *expected* patterns are reasonably stable from year-to-year
  - Since claim development studies are time-consuming, it's best to perform the analysis in the third or fourth quarter of the year (to construct models before the year-end time crunch)
  - Once models are created, they can be applied to year-end and quarter-end claims and exposures to estimate IBNR

#### IV. Claim Report and Payment Lags

- ◇ When analyzing reinsurance development patterns, it's useful to consider the inverse of the usual chain-ladder development factors. These are known as lags
- ◇ Lags can then be used to create a graph with time in years on the x-axis and the lag percentage on the y-axis
- ◇ This graph resembles a probability cdf and can be interpreted as the probability that any particular claims dollar will be reported to the reinsurer by time  $t$
- ◇ Statistics can be calculated from this curve (such as expected value and standard deviation) to compare one claim pattern to another

#### V. Methods for Short-Tailed Exposure Categories

- ◇ Any exposure for which losses are reported and settled quickly
- ◇ Consists of:
  - Treaty property proportional
  - Treaty property catastrophe
  - Treaty property excess
  - Facultative property
- ◇ Method 1: Set IBNR equal to some percentage of the latest-year EP
- ◇ Method 2: Reserve up to a selected loss ratio (especially for new LOBs), where the selected loss ratio is larger than the one computed from reported non-cat claims
- ◇ If losses are summarized by underwriting year, then percentage estimates should be used to allocate losses to true accident years to avoid overstating accident year loss development

#### VI. Methods for Medium-Tailed Exposure Categories

- ◇ Any exposure for which claims are almost completely settled within five years and with average aggregate claims dollar report lag of one to two years
- ◇ Consists of:
  - Treaty property excess higher layers (takes a long time to penetrate the layer)
  - Construction risks (discovery period can be long)
  - Surety (salvage recoveries tend to have long tail)

- ◇ Method 1: Standard chain-ladder (CL) method
  - Advantage is that it strongly correlates future development with an overall lag pattern and with the claims reported for each accident year
  - Disadvantage is that IBNR is so correlated with reported claims that estimates are not very credible for recent, immature years
  - Possible to use paid losses rather than reported losses to increase stability, but that could backfire for immature years where very few losses have been paid

## VII. Methods for Long-Tailed Exposure Categories

- ◇ Any exposure for which the average aggregate claims dollar report lag is over two years and whose claims are not settled for many years
- ◇ Consists of:
  - Treaty casualty excess (longest lags excluding asbestos, pollution, etc.)
  - Treaty casualty proportional
  - Asbestos, pollution, etc.
- ◇ First step is to separate these exposures into more homogeneous groups based on guidance from marketing, underwriting, claims and accounting personnel
- ◇ **Asbestos, pollution, other health hazard and other mass tort**
  - Must be analyzed separately
  - No claims for long time periods followed by gigantic claims
  - Cannot use traditional reserving methods
  - Must rely on complex reserving models
- ◇ Method 1: Standard chain-ladder method (not great for immature years)
- ◇ Method 2: Bornhuetter/Ferguson (BF) method
  - Advantage is that it correlates future development for each year with an exposure measure (the reinsurance premium multiplied by a selected loss ratio)
  - Disadvantages with the BF IBNR estimate include that it is dependent upon the selected loss ratio and that the estimate ignores reported claims for each accident year

◇ **Method 3: Stanard-Bühlmann (Cape Cod) method**

- **Key innovation of the SB method is that the ultimate expected loss ratio for all years combined is estimated from the overall reported claims experience, instead of being selected judgmentally, as in the BF method**
- A **disadvantage** of the SB method is that the IBNR by year is dependent upon rate-level adjusted premium
- Before moving on to the formulas, let's define a few terms:
  - ◇  $SBELR$  = SB estimate of the ELR
  - ◇  $SBIBNR(k)$  = SB IBNR estimate, year  $k$
  - ◇  $RRL(k)$  = reported reinsurance loss, year  $k$
  - ◇  $ARPP(k)$  = adjusted risk pure premium, year  $k$
  - ◇  $Rlag(k)$  = aggregate claim dollar report lag, year  $k$
- Using the terms above:

$$SBELR = \frac{\sum RRL(k)}{\sum [ARPP(k) \cdot Rlag(k)]}$$

$$SBIBNR(k) = SBELR \cdot ARPP(k) \cdot (1 - Rlag(k))$$

- The term  $ARPP(k) \cdot Rlag(k)$  is also known as the “used-up premium” for year  $k$
- Let's look at an example. Given the following as of December 31, 2014:

AY	Earned Risk Pure Premium	Adjusted Premium	Aggregated Reported Loss	Aggregate Loss Report Lag
2012	1000	1200	800	0.70
2013	1500	1800	500	0.40
2014	2000	2000	400	0.30

- First, calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2012	1200	0.70	$840 = 1200(0.70)$
2013	1800	0.40	720
2014	2000	0.30	600



- Next, we calculate the SB ELR as follows:

$$\diamond \text{ SB ELR} = \text{reported losses} / \text{used-up premium} = (400 + 500 + 800) / (600 + 720 + 840) = 0.787$$

- Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
2012	1200	0.70	$283.32 = 1200(0.787)(1 - 0.70)$
2013	1800	0.40	849.96
2014	2000	0.30	1101.80
Total			<b>2235.08</b>

#### ◇ Method 4: Simple credibility IBNR estimate

- When we don't have complete confidence in rate-level premium adjustments, we can weight the CL and SB methods together using a credibility factor
- Gives more weight to the SB estimate for immature years
- Gives more weight to the CL estimate for older years where the cumulative rate-level adjustments are less reliable
- Before moving on the formula, let's define a few terms:

$$\diamond \text{ CredIBNR}(k) = \text{credibility IBNR for year } k$$

$$\diamond \text{ CLIBNR}(k) = \text{CL IBNR for year } k$$

$$\diamond \text{ CF} = \text{credibility factor (between 0 and 1)}$$

$$\diamond \text{ Z}(k) = \text{CF} \cdot \text{Rlag}(k)$$

- Using the terms above:

$$\boxed{\text{CredIBNR}(k) = \text{Z}(k) \cdot \text{CLIBNR}(k) + (1 - \text{Z}(k)) \cdot \text{SBIBNR}(k)}$$

- Given a credibility factor of 0.6, let's revisit our example from earlier:

AY	Reported Losses	Aggregate Loss Report Lag	SB IBNR	CL IBNR	Cred IBNR
2012	800	0.70	283.32	342.86	308.33
2013	500	0.40	849.96	750.00	825.97
2014	400	0.30	1101.80	933.33	1071.48
Total			2235.08	2026.19	<b>2205.78</b>

- Here are the calculations for AY 2012:
  - ◊  $CL\ IBNR = 342.86 = \frac{800}{0.70} - 800$
  - ◊  $Cred\ IBNR = 308.33 = 0.60(0.70)(342.86) + (1 - 0.60(0.70))(283.32)$
- ◊ Method 5: Other credibility procedures
  - Weight together IBNR estimates based on reported claims and paid claims
    - ◊ Reported claims include case reserves that vary over time depending on the claims adjuster setting the reserve
    - ◊ Paid claims tend to be more stable, assuming you have sufficient data and believe the data to have a consistent expected payment pattern
    - ◊ Weights could be based on relative claim report and payment lags for each year
  - Use the ELR inherent in the underlying pricing of the exposure in lieu of or in conjunction with the SB ELR
    - ◊ We call these *a priori* ELR estimates and can use them as our BF ELR estimates. These can then be used to calculate BF IBNR
    - ◊ Weight this *a priori* IBNR against the CL IBNR (Bengtander Method)
- ◊ Method 6: Alternative estimation methodologies
  - Stochastic reserving models
    - ◊ An advantage is that they give us more information and provide insight not available with traditional methods
    - ◊ A disadvantage is that they tend to be complicated and black boxish, making them difficult to interpret and explain to management
  - Claim count/claim severity model
    - ◊ An advantage is that we can build models for various lag distributions, and then connect them with models for the dollar reserving and the payments on individual claims

## VIII. Monitoring and Testing Predictions

- ◊ Monitoring and testing quarterly claims run-off against predictions provides early warning of problems
- ◊ For short and medium-tailed lines, past AY run-off can be compared with previous year-end reported open reserves and IBNR reserves

- ◇ More sophistication required for long-tailed lines
- ◇ If more claims emerge than what was expected, what does it mean?
  - Is it purely random?
  - Does it indicate that the beginning IBNR was too small?
  - Were the lags too short?
- ◇ Continue to monitor claims each quarter to see if pattern persists. If so, we may need to lengthen the lags

## **IX. Final Comments**

- ◇ Differences between primary company loss reserving and reinsurance reserving:
  - Less information
  - Longer report and settlement timing delays
  - Low frequency and high severity claims

## Original Mathematical Problems & Solutions

### MP #1

Given the following as of December 31, 2012:

AY	Earned Risk Pure Premium	Adjusted Premium	Aggregated Reported Loss	Aggregate Loss Report Lag
2010	\$500	\$800	375	0.70
2011	800	900	250	0.40
2012	1,000	1,000	200	0.30

Estimate the total IBNR using the Stanard-Bühlmann method.

**Solution:**

◇ First, calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2010	800	0.70	$560 = 800(0.70)$
2011	900	0.40	360
2012	1000	0.30	300

◇ Next, calculate the SB ELR:

- $SB\ ELR = \text{reported losses} / \text{used-up premium} = (375 + 250 + 200) / (560 + 360 + 300) = 0.676$

◇ Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
2010	800	0.70	$162.24 = 800(0.676)(1 - 0.70)$
2011	900	0.40	365.04
2012	1000	0.30	473.20
Total			\$1,000.48

**MP #2**

Given the following:

AY	Adjusted Premium	Claims at 12/31/12	Claim Report Lag at 12/31/12	Claim Report Lag at 6/30/13	Actual Claims at 6/30/13
2008	\$2,700	\$2,500	0.95	0.98	\$2,500
2009	3,000	2,100	0.85	0.90	2,310
2010	4,200	1,500	0.70	0.77	1,800
2011	4,000	1,100	0.50	0.58	1,265
2012	6,000	2,400	0.30	0.41	3,060

- a) Using the chain-ladder method, calculate the difference between the actual claims and the expected claims at 6/30/13. Give two possible reasons for the difference.
- b) Using the credibility method with a credibility factor of 0.4, calculate the difference between the actual claims and the expected claims at 6/30/13. Give two possible reasons for the difference.

**Solution to part a:**

◇ First, calculate the CL IBNR as of 12/31/12:

AY	CL IBNR
2008	$131.579 = (2500/.95) - 2500$
2009	370.588
2010	642.857
2011	1100
2012	5600

◇ Next, calculate the expected claims at 6/30/13:

AY	Expected Claims at 6/30/13
2008	$2578.947 = 2500 + 131.579 \left( \frac{0.98-0.95}{1-0.95} \right)$
2009	2223.529
2010	1650
2011	1276
2012	3280

◇ Calculate the difference between the actual claims and the expected claims at 6/30/13:

AY	Expected Claims at 6/30/13	Actual Claims at 6/30/13	Difference
2008	2578.947	2500	$-78.947 = 2500 - 2578.947$
2009	2223.529	2310	86.471
2010	1650	1800	150
2011	1276	1265	-11
2012	3280	3060	-220
Total			<span style="border: 1px solid black; padding: 2px;">-\$73.48</span>

◇ **Two possible reasons for the difference are that the beginning IBNR was too large OR that the report lags were too long**

**Solution to part b:**

◇ First, calculate the SB ELR as of 12/31/12:

AY	Used-Up Premium	Claims at 12/31/12
2008	$2565 = 2700(0.95)$	2500
2009	2550	2100
2010	2940	1500
2011	2000	1100
2012	1800	2400
Total	11855	9600

Thus, the SB ELR =  $9600/11855 = 0.810$

◇ Next, calculate the SB IBNR as of 12/31/12:

AY	SB IBNR
2008	$109.35 = 0.810(2700)(1 - 0.95)$
2009	364.50
2010	1020.60
2011	1620
2012	3402

◇ Next, calculate the Credibility IBNR as of 12/31/12:

AY	CF	$Z(k)$	Credibility IBNR
2008	0.4	$0.38 = 0.4(0.95)$	$117.797 = 131.579(0.38) + 109.35(1 - 0.38)$
2009	0.4	0.34	366.570
2010	0.4	0.28	914.832
2011	0.4	0.20	1516
2012	0.4	0.12	3665.76



◇ Next, calculate the expected claims at 6/30/13:

AY	Expected Claims at 6/30/13
2008	$2570.678 = 2500 + 117.797 \left( \frac{0.98-0.95}{1-0.95} \right)$
2009	2222.19
2010	1713.461
2011	1342.56
2012	2976.048

◇ Calculate the difference between the actual claims and the expected claims at 6/30/13:

AY	Expected Claims at 6/30/13	Actual Claims at 6/30/13	Difference
2008	2570.678	2500	$-70.678 = 2500 - 2570.678$
2009	2222.19	2310	87.81
2010	1713.461	1800	86.539
2011	1342.56	1265	-77.56
2012	2976.048	3060	83.952
Total			\$110.06

◇ **Two possible reasons for the difference are that the beginning IBNR was too small OR that the report lags were too short**

## MP #3

A reinsurance company has underwritten four proportional casualty treaties. Given the following for the treaties as of December 31, 2012:

Treaty ID	Inception Date	Treaty Term	Treaty Coverage Basis	Written Premium	Aggregate Reported Loss
A	Jan. 1, 2008	1 Year	Loss Occuring	\$2,000	\$3,000
B	Oct. 1, 2008	0.5 Year	Policies Written	4,000	3,000
C	Jan. 1, 2009	1 Year	Loss Occuring	4,000	2,500
D	Jul. 1, 2010	1 Year	Policies Written	5,000	1,500

Age of Accident Year (in months)	Age to Ultimate LDF
12	2.50
24	1.60
36	1.25
48	1.10
60	1.05

- ◇ All underlying policies are one-year, occurrence-based policies with effective dates uniformly distributed over the year
- ◇ Treaty A commission, brokerage and internal expense is 10% of premium
- ◇ Treaty B commission, brokerage and internal expense is 10% of premium
- ◇ Treaty C commission, brokerage and internal expense is 5% of premium
- ◇ Treaty D commission, brokerage and internal expense is 20% of premium
- ◇ The loss development factors apply to all exposures in the applicable accident year

Calculate the IBNR for the four treaties combined using the Stanard-Bühlmann method.

**Solution:**

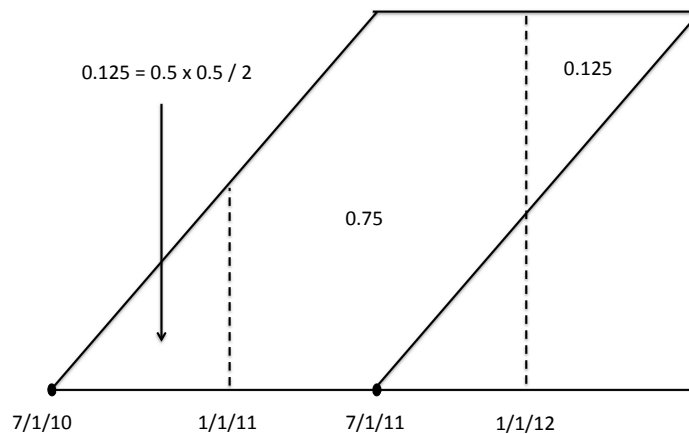
◇ In order to use the Stanard-Bühlmann method, we need to convert the Written Premium into Earned Risk Pure Premium

- Since all policies are fully earned by 12/31/12, written premium = earned premium
- To convert the earned premium into earned risk pure premium, we multiply the premium by one minus the expenses for each treaty. This gives us the following:

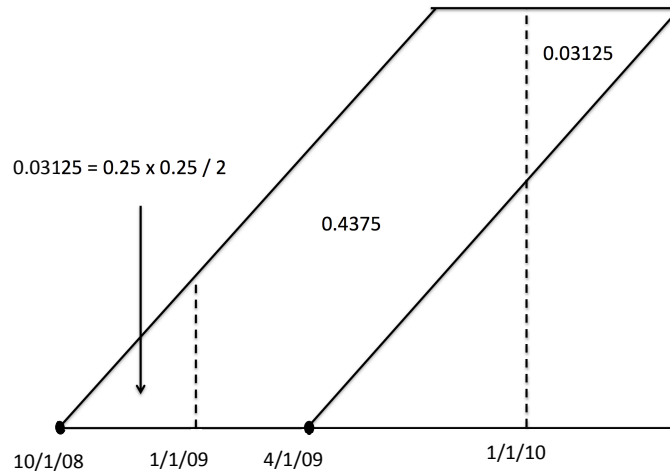
Treaty ID	Earned Risk Pure Premium
A	$1800 = 2000(1 - 0.1)$
B	3600
C	3800
D	4000

◇ The trickiest part of this problem is the allocation of the treaties to accident years. After that, it follows exactly like a typical Stanard-Bühlmann problem. Let's look at the allocation:

- Since Treaty A and C are one-year, loss occurring policies with 1/1/XX effective dates, they align exactly with accident years 2008 and 2009. No further allocation is required
- Treaties B and D are tougher since they are on a policies written basis (i.e. underwriting year). Fortunately, we are told that the underlying policies are uniformly distributed over the year. This means we can use the parallelogram method to allocate the treaties to the appropriate accident years
- Starting with Treaty D:



- Treaty B is tougher:



- Notice that the total area of the Treaty B parallelogram is only 0.5. Since the treaty term is 0.5 years and the underlying policies are one-year policies, we end up with a parallelogram that is exactly half of the Treaty D parallelogram

◇ We can now allocate the treaties to accident years:

Treaty ID	AY Allocation:				
	2008	2009	2010	2011	2012
A	1	0	0	0	0
B	$0.0625 = 0.03125/0.5$	0.875	0.0625	0	0
C	0	1	0	0	0
D	0	0	0.125	0.75	0.125

◇ Next, organize our data by AY using the weights above:

AY	Earned Risk	Aggregate
	Pure Premium	Reported Loss
2008	$2025 = 1800(1) + 0.0625(3600)$	$3187.50 = 3000(1) + 0.0625(3000)$
2009	6950	5125
2010	725	375
2011	3000	1125
2012	500	187.50

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◇ Calculate the SB ELR:

AY	Earned Risk Pure Premium	Aggregate Reported Loss	Report Lag	Used-Up Premium
2008	2025	3187.50	$0.952 = 1/1.05$	$1927.80 = 2025(0.952)$
2009	6950	5125	0.909	6317.55
2010	725	375	0.8	580
2011	3000	1125	0.625	1875
2012	500	187.50	0.4	200
Total		10000		10900.35

Thus, the SB ELR =  $10000/10900.35 = 0.917$

◇ Finally, calculate the SB IBNR:

AY	SB IBNR
2008	$89.132 = 0.917(2025)(1 - 0.952)$
2009	579.957
2010	132.965
2011	1031.625
2012	275.10
Total	<span style="border: 1px solid black; padding: 2px;">\$2,108.78</span>

## Original Essay Problems

### EP #1

Briefly describe the six components of a reinsurer's loss reserve.

### EP #2

Identify the three steps involved in reinsurer loss reserving.

### EP #3

- a) Define claim report lag  $R(t)$  in terms of the standard chain-ladder age-to-ultimate development factor.
- b) Explain how the claim report lag can be interpreted as a probability cumulative distribution function. Give one reason why this interpretation is useful.

### EP #4

Given the following exposure types:

- ◇ Short-tailed exposures
- ◇ Medium-tailed exposures
- ◇ Long-tailed exposures

- a) Identify one loss reserve estimation method for each exposure type above.
- b) Provide two examples of each exposure type above.

### EP #5

Patrik describes a credibility procedure that weights the chain-ladder IBNR with the Stanard-Bühlmann IBNR.

- a) Briefly explain the rationale behind this procedure.
- b) Describe two alternative credibility procedures.

## Original Essay Solutions

### ES #1

- ◇ Component 1: Case reserves reported by the ceding companies
  - Reported on an individual claim basis (excess contracts) or in bulk summary form (proportional contracts)
- ◇ Component 2: Reinsurer additional reserves on individual claims
  - Reinsurer reviews individual claims and specifies additional case reserves if necessary
- ◇ Component 3: Actuarial estimate of future development on components 1 and 2
  - Known as IBNER (incurred but not enough reserved)
- ◇ Component 4: Actuarial estimate of pure IBNR
  - Usually combined with component 3 due to limitations in data systems. Together, components 3 and 4 are known as IBNR
- ◇ Component 5: Discount for future investment income
  - Companies can take credit for future investment income on assets supporting certain types of claims, such as WC permanent total cases, auto PIP annuity claims and medical professional liability claims
- ◇ Component 6: Risk load
  - Adverse deviation loading is used to keep reserves at a conservative level

### ES #2

- ◇ Partition the reinsurance portfolio into reasonably homogeneous exposure groups that are relatively consistent over time with respect to mix of business
- ◇ Analyze the historical development patterns. If possible, consider individual case reserve development and the emergence of IBNR claims separately
- ◇ Estimate the future development. If possible, estimate the bulk reserves for IBNER and pure IBNR separately

**ES #3**

Part a:

- ◇ The claim report lag at time  $t$  is the inverse of the chain-ladder age-to-ultimate development factor

Part b:

- ◇ The claim report lag can be read as the probability that any particular claims dollar will be reported to the reinsurer by time  $t$ . This view allows us to compute statistics of the claims reporting process, enabling us to compare one claim report pattern with another

**ES #4**

Part a:

- ◇ Short-tailed – set IBNR equal to some percentage of the latest-year earned premium
- ◇ Medium-tailed – chain-ladder method
- ◇ Long-tailed – Cape Cod method

Part b:

- ◇ Short-tailed – treaty property proportional, treaty property catastrophe
- ◇ Medium-tailed – treaty property excess higher layers, construction risk
- ◇ Long-tailed – treaty casualty excess, asbestos

**ES #5**

Part a:

- ◇ Due to the difficulty in obtaining rate-level adjusted premium, we may not have complete confidence in the Stanard-Bühlmann approach. Thus, we can weight the chain-ladder and Stanard-Bühlmann methods together using a credibility factor

Part b:

- ◇ Weight together IBNR estimates based on reported claims and paid claims. The weights could be based on relative claim report and payment lags for each year



- ◇ Use the ELR inherent in the underlying pricing of the exposure in lieu of or in conjunction with the Stanard-Bühlmann ELR. This ELR can be used to calculate BF IBNR. We then weight this BF IBNR with the chain-ladder IBNR

## Past CAS Exam Problems & Solutions

### E7 2014 #13

Describe four aspects of reinsurance loss reserving that make it somewhat more difficult than primary loss reserving.

**Solution:**

- ◇ Longer development pattern than primary insurer due to:
  - Extended by cedant's reporting pipeline
  - Cedants tend to under-reserve large claims
  - Extreme delays in searching and reporting latent claims
- ◇ Consistent upward development, due to:
  - Inflation impact
  - Cedants tend to under-reserve ALAE
  - Cedants tend to initially reserve large claims at modal values
- ◇ Industry data is not helpful due to:
  - No breakdown of reinsurer's exposures into homogeneous groups
  - Severity of development increases with attachment point
- ◇ Claim development is extremely different due to:
  - Reinsurance contracts are unique
  - Significant fluctuation during development because of single large claims

**E7 2014 #14**

Given the following reinsurance company data (\$000) evaluated as of December 31, 2013:

Calendar/ Accident Year	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss
2010	5,000	5,000	3,000
2011	4,500	5,500	2,500
2012	5,000	6,000	2,500
2013	6,500	6,500	1,500
Total	20,000	23,000	9,500

## Reported Loss Development

## Factors to Ultimate

12 months	2.50
24 months	2.00
36 months	1.60
48 months	1.25
60 months	1.00

- ◇ Adjusted premium incorporates changes in primary rates, underwriting, and exposure affecting the loss potential

Calculate the Stanard-Bühlmann (Cape Cod) ultimate loss ratio for 2011.

**Solution:**

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2010	5000	$0.800 = \frac{1}{1.25}$	$4000 = 5000(0.80)$
2011	5500	0.625	3437.5
2012	6000	0.500	3000
2013	6500	0.400	2600

◇ Calculate the SB ELR:

- $SB\ ELR = \frac{\text{reported losses}}{\text{used-up premium}} = \frac{9500}{4000+3437.5+3000+2600} = 0.729$

◇ Calculate the SB IBNR for AY 2011:

- $SB\ IBNR = 5500(0.729)(1 - 0.625) = 1503.5625$

◇ Calculate the ultimate loss ratio for AY 2011:

- $Ultimate\ loss = \text{reported loss} + SB\ IBNR = 2500 + 1503.5625 = 4003.5625$

- $Ultimate\ loss\ ratio = \frac{\text{ultimate losses}}{\text{earned premium}} = \frac{4003.5625}{4500} = \boxed{0.89}$

**E7 2013 #10**

Given the following information (\$000) as of December 31, 2012:

Accident Year	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss	Aggregate Loss Report Lag
2010	9,200	9,200	3,600	70%
2011	8,500	7,400	2,500	50%
2012	10,000	10,000	4,200	30%

- Use the Stanard-Bühlmann method to estimate the IBNR for accident years 2010 through 2012 combined.
- Use a credibilty-weighted combination of the Stanard-Bühlmann and chain-ladder estimates to calculate a total IBNR estimate for accident years 2010 through 2012 combined. Assume credibility is a linear function of the report lag with a credibility factor of 0.35.
- Briefly describe one advantage and one disadvantage of the Stanard-Bühlmann method as compared to the chain-ladder method.

**Solution to part a:**

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2010	9200	0.70	$6440 = 9200(0.70)$
2011	7400	0.50	3700
2012	10000	0.30	3000

◇ Calculate the SB ELR:

$$\bullet \text{ SB ELR} = \frac{\text{reported losses}}{\text{used-up premium}} = \frac{3600+2500+4200}{6440+3700+3000} = 0.784$$

◇ Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
2010	9200	0.70	$2163.84 = 9200(0.784)(1 - 0.70)$
2011	7400	0.50	2900.80
2012	10000	0.30	5488.00
Total			\$10,552.64

**Solution to part b:**

AY	CL IBNR	Z	Cred IBNR
2010	$1542.857 = 3600(\frac{1}{0.70} - 1)$	$0.245 = 0.35(0.70)$	$2011.699 = 1542.857(0.245) + 2163.840(1 - 0.245)$
2011	2500	0.175	2830.660
2012	9800	0.105	5940.760
Total			\$10,783.12

**Solution to part c:**

- ◇ Advantage – more stable in most recent accident years
- ◇ Disadvantage – requires rate-adjusted earned premium which may be difficult to obtain

**E7 2012 #10**

Given the following information for two proportional casualty treaties as of December 31, 2011:

Treaty ID	Inception Date	Treaty Coverage Basis	Written Premium	Aggregate Reported Loss
A	Jan. 1, 2009	Loss Occurring	\$80,000	\$50,000
B	Jul. 1, 2009	Policies Written	100,000	20,000

Age of Accident Year (in months)	Age to Ultimate LDF
36	1.25
24	2.00
12	3.00

- ◇ Both treaties have a one-year term
  - ◇ All underlying policies are one-year, occurrence-based policies with effective dates uniformly distributed over the year
  - ◇ Treaty A commission, brokerage and internal expense is 5% of premium
  - ◇ Treaty B commission, brokerage and internal expense is 20% of premium
  - ◇ The loss development factors apply to all exposures in the applicable accident year
- a) Estimate the ultimate losses for the two treaties combined using the Stanard-Bühlmann method.
  - b) Identify a situation in which the chain-ladder method is preferred over the Stanard-Bühlmann method when estimating IBNR for a reinsurer with long-tail exposures.
  - c) Identify a situation in which the Stanard-Bühlmann method is preferred over the chain-ladder method when estimating IBNR for a reinsurer with long-tail exposures.
  - d) Identify the key innovation of the Stanard-Bühlmann method over the Bornhuetter/Ferguson method.



**Solution to part a:**

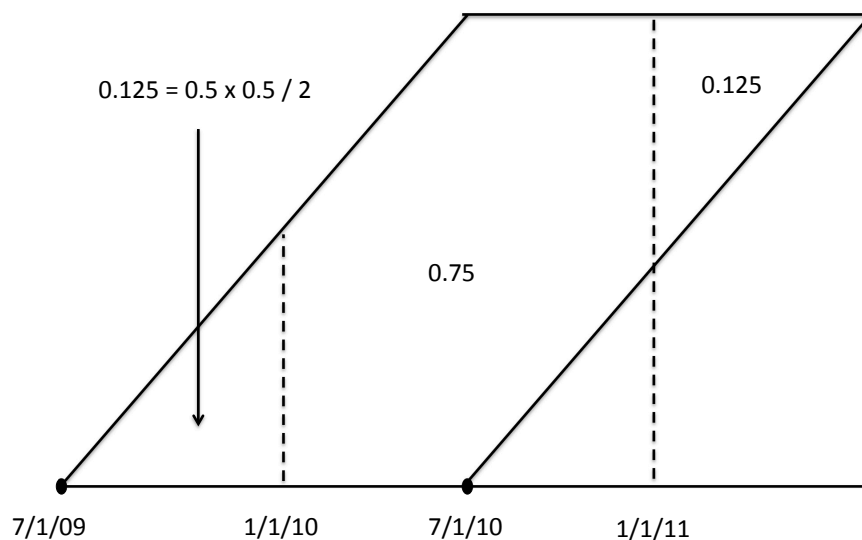
◇ In order to use the Stanard-Bühlmann method, we need to convert the Written Premium into Earned Risk Pure Premium

- Since all policies are fully earned by 12/31/11, written premium = earned premium
- To convert the earned premium into earned risk pure premium, we multiply the premium by one minus the expenses for each treaty. This gives us the following:

Treaty ID	Earned Risk Pure Premium
A	$76000 = 80000(1 - 0.05)$
B	80000

◇ The trickiest part of this problem is the allocation of the treaties to accident years. After that, it follows exactly like a typical Stanard-Bühlmann problem. Let's look at the allocation:

- Since Treaty A is a one-year, loss occurring policy with a 1/1/09 effective date, it aligns exactly with accident year 2009. No further allocation is required
- Treaty B is tougher since it is on a policies written basis (i.e. underwriting year). Fortunately, we are told that the underlying policies are uniformly distributed over the year. This means we can use the parallelogram method to allocate the treaty to the appropriate accident years



- We can now allocate the treaties to accident years:

Treaty ID	AY Allocation:		
	2009	2010	2011
A	1	0	0
B	0.125	0.75	0.125

- Next, organize our data by AY using the weights above:

AY	Earned Risk Pure Premium	Aggregate Reported Loss
2009	$86000 = 76000(1) + 0.125(80000)$	$52500 = 50000(1) + 0.125(20000)$
2010	60000	15000
2011	10000	2500

- Calculate the SB ELR:

AY	Earned Risk Pure Premium	Aggregate Reported Loss	Report Lag	Used-Up Premium
2009	86000	52500	$0.80 = \frac{1}{1.25}$	$68800 = 0.80(86000)$
2010	60000	15000	0.500	30000
2011	10000	2500	0.333	3330

Thus, the SB ELR =  $\frac{52500+15000+2500}{68800+30000+3330} = 0.685$

- Finally, calculate the SB ultimate losses:

AY	SB Ultimate
2009	$64282 = 52500 + 86000(0.685)(1 - 0.80)$
2010	35550
2011	7068.95
Total	$\$106,900.95$

**Solution to part b:**

- ◇ The chain-ladder method is preferred when it is difficult to obtain adjusted on-level premium

**Solution to part c:**

- ◇ The Stanard-Bühlmann method is preferred when there are fluctuations in reported losses

**Solution to part d:**

- ◇ The expected loss ratio is estimated from the overall reported claims experience, instead of being selected judgmentally, as in the BF method

**E7 2012 #11**

Identify and explain five technical problems that make loss reserving for a reinsurer more difficult than loss reserving for a primary company.

**Solution:**

- ◇ Longer development pattern than primary insurer due to:
  - Extended by cedant's reporting pipeline
  - Cedants tend to under-reserve large claims
  - Extreme delays in searching and reporting latent claims
- ◇ Consistent upward development, due to:
  - Inflation impact
  - Cedants tend to under-reserve ALAE
  - Cedants tend to initially reserve large claims at modal values
- ◇ Industry data is not helpful due to:
  - No breakdown of reinsurer's exposures into homogeneous groups
  - Severity of development increases with attachment point
- ◇ Missing claim information
  - Reinsurer's exposure is not completely measured in most recent year
  - Missing detailed claim information on excess loss level
- ◇ Claim development is extremely different due to:
  - Reinsurance contracts are unique
  - Significant fluctuation during development because of single large claims

**E7 2011 #10**

Given the following information for a line of business for a reinsurance company as of December 31, 2010:

Calendar Accident Year	Earned	Adjusted Premium (\$000)	Aggregate	Aggregate Loss Report Lag	Chain-
	Risk Pure Premium (\$000)		Reported Loss (\$000)		Ladder IBNR (\$000)
2005	9,200	10,000	9,500	95%	500
2006	10,100	12,000	8,000	85%	1,412
2007	11,500	11,500	7,300	72%	2,839
2008	12,300	13,000	6,500	55%	5,318
2009	13,500	14,100	4,800	40%	7,200
2010	15,000	15,000	2,500	28%	6,429
Total	71,600	75,600	38,600		23,698

- Use the Stanard-Bühlmann method to estimate the total IBNR.
- Briefly describe one advantage and one disadvantage of using the Stanard-Bühlmann method to calculate IBNR for a reinsurer.
- Use a credibility-weighted combination of the Stanard-Bühlmann and chain-ladder estimates with a linear function of the report lag as the credibility weight to calculate a total IBNR estimate. For the chain-ladder estimate, use a credibility factor of 0.6.

**Solution to part a:**

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2005	10000	0.95	9500 = 10000(0.95)
2006	12000	0.85	10200
2007	11500	0.72	8280
2008	13000	0.55	7150
2009	14100	0.40	5640
2010	15000	0.28	4200

◇ Calculate the SB ELR:

$$\bullet \text{ SB ELR} = \frac{\text{reported losses}}{\text{used-up premium}} = \frac{9500+8000+7300+6500+4800+2500}{9500+10200+8280+7150+5640+4200} = 0.858$$

◇ Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
2005	10000	0.95	429.00 = 10000(0.858)(1 - 0.95)
2006	12000	0.85	1544.40
2007	11500	0.72	2762.76
2008	13000	0.55	5019.30
2009	14100	0.40	7258.68
2010	15000	0.28	9266.40
Total			\$26,280.54

**Solution to part b:**

- ◇ Advantage – Uses actual claims data to derive ELR rather than judgmentally selecting it
- ◇ Disadvantage – Difficult to obtain accurate on-level premium data

## Solution to part c:

AY	Z	Cred IBNR
2005	$0.570 = 0.60(0.95)$	$469.470 = 500(0.57) + 429(1 - 0.57)$
2006	0.510	1476.876
2007	0.432	2795.697
2008	0.330	5117.871
2009	0.240	7244.597
2010	0.168	8789.717
Total		\$25,894.23





**E7 2011 #11**

- a) Describe three reasons why claim report lags for reinsurers tend to be longer than for primary insurers.
- b) Describe two reasons why reported losses for reinsurers generally develop more than for primary insurers after the claims have been reported.
- c) Briefly describe three reasons why many standard actuarial loss development methods used by primary insurers may not work as well for reinsurers.
- d) Briefly describe two reasons why industry loss development statistics may not be useful for estimating loss development patterns for a reinsurer.

**Solution to part a:**

- ◇ Longer reporting pipeline (reported to cedant, to reinsurance department, to intermediary, to reinsurer)
- ◇ Lengthened by undervaluation of serious claims by cedant
- ◇ Some mass tort claims have extreme delays in discovery; loss dates may be decided by courts

**Solution to part b:**

- ◇ Tendency to under-reserve allocated loss adjustment expenses. Thus, the ultimate loss is understated
- ◇ Claims adjusters tend to reserve at modal values. Thus, claims may not immediately pierce the attachment point of the reinsurance treaty

**Solution to part c:**

- ◇ Exposures assumed by reinsurance companies tend to be heterogeneous. This is a problem because traditional reserving methods require homogeneous data
- ◇ Low claim frequencies and long report lags cause extreme fluctuation in historical data
- ◇ Reinsurers know much less about specific exposures being covered than a primary carrier

**Solution to part d:**

- ◇ Information is much less specific
- ◇ Longer claim report

**E6 2010 #17**

Given the following information for a reinsurance contract as of December 31, 2009:

Calendar/ Accident Year	Earned Risk Pure Premium	On-level Premium	Chain- Ladder IBNR	Stanard- Bühlmann IBNR
2008	\$17,000,000	\$16,000,000	\$3,000,000	\$3,250,000

Age Interval in Months	Reported Loss Development Factor
12-24	1.35
24-36	1.15
36-48	1.08
48-60	1.03
60-ult.	1.00

- Determine the aggregate report lag as of December 31, 2009.
- Use Patrik's method with credibility factor 0.4 to calculate the credibility-weighted IBNR estimate as of December 31, 2009.

**Solution to part a:**

$$\diamond \text{Lag} = \frac{1}{\text{LDF}} = \frac{1}{1.15(1.08)(1.03)} = \boxed{0.782}$$

**Solution to part b:**

$$\diamond Z = 0.782(0.4) = 0.313$$

$$\diamond \text{Cred IBNR} = 3(0.313) + 3.25(1 - 0.313) = \boxed{3.172\text{M}}$$

**E6 2010 #18**

Given the following data for a reinsurer as of December 31, 2009:

Calendar/ Accident Year	Earned Premium	Adjusted Premium	Aggregate Reported Loss	Age-to- Ultimate LDF
2005	\$10,000	\$12,000	\$9,000	1.03
2006	11,000	12,000	9,000	1.11
2007	13,000	13,000	7,000	1.25
2008	15,000	14,000	10,000	1.47
2009	17,000	15,000	6,000	2.00

- a) Use the Stanard-Bühlmann method to calculate the IBNR for accident year 2008 as of December 31, 2009.
- b) Discuss two problems that may affect the accuracy of a reinsurer's earned premium data.

**Solution to part a:**

- ◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2005	12000	$0.971 = \frac{1}{1.03}$	$11652 = 12000(0.971)$
2006	12000	0.901	10812
2007	13000	0.800	10400
2008	14000	0.680	9520
2009	15000	0.500	7500

- ◇ Calculate the SB ELR:

- $SB\ ELR = \frac{\text{reported losses}}{\text{used-up premium}} = \frac{9000+9000+7000+10000+6000}{11652+10812+10400+9520+7500} = 0.822$

- ◇ Calculate the AY 2008 SB IBNR:

- $SB\ IBNR = 14000(0.822)(1 - 0.680) = \boxed{\$3,682.56}$

**Solution to part b:**

- ◇ Since the reinsurer relies on the insurer to report premium, there is often a lag in reporting
- ◇ Since earned premium is often reported in aggregate to the reinsurer, the reinsurer must make assumptions to split the premium by LOB

**E6 2009 #31**

Given the following as of December 31, 2008:

Calendar/ Accident Year	Earned Risk Pure Premium	Adjusted Premium	Aggregate Reported Loss	Aggregate Loss Report Lag
2006	\$8,000	\$10,000	\$7,500	90%
2007	8,500	10,000	4,500	60%
2008	10,000	10,000	1,125	25%
Total	26,500	30,000	13,125	

Calculate the Stanard-Bühlmann IBNR estimate for all years combined.



**Solution:**

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2006	10000	0.90	9000 = 10000(0.90)
2007	10000	0.60	6000
2008	10000	0.25	2500

◇ Calculate the SB ELR:

$$\bullet \text{ SB ELR} = \frac{\text{reported losses}}{\text{used-up premium}} = \frac{7500+4500+1125}{9000+6000+2500} = 0.75$$

◇ Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
2006	10000	0.90	750 = 10000(0.75)(1 - 0.90)
2007	10000	0.60	3000
2008	10000	0.25	5626
Total			\$9,375

**E6 2009 #32**

Reinsurers are encouraged to partition data received from cedants into reasonably homogeneous exposure categories.

Given the following historical information for a company that reinsures workers compensation and general liability for three primary insurance companies:

Type of Contract	Primary Line of Business	Cedant	Aggregate Number of Claims for Accident Years 2006-2008
Facultative	Workers Compensation	Cedant 1	12,123
Facultative	Workers Compensation	Cedant 2	50,178
Facultative	Workers Compensation	Cedant 3	6
Facultative	General Liability	Cedant 1	8,799
Facultative	General Liability	Cedant 2	25,472
Facultative	General Liability	Cedant 3	17
Treaty	Workers Compensation	Cedant 1	21,947
Treaty	Workers Compensation	Cedant 2	38,076
Treaty	Workers Compensation	Cedant 3	9
Treaty	General Liability	Cedant 1	2,157
Treaty	General Liability	Cedant 2	12,972
Treaty	General Liability	Cedant 3	48

◇ The reinsurer began writing contracts in 2006

- a) Discuss how the reinsurer should partition this data before analyzing historical development patterns.
- b) In general, identify two actions a reinsurer could take to help determine which variables to use for partitioning reinsurance data.

**Solution to part a:**

- ◇ The reinsurer should separate Facultative and Treaty contracts since they probably have different risk characteristics. Workers Compensation and General Liability should also be separated since they may have different payment patterns. Due to the lack of data, the three cedants can be combined in the analysis

**Solution to part b:**

- ◇ Talk to the Claims department and ask if the variable affects their claims payment history
- ◇ Talk to Underwriting and ask how different the risks are between the groups

**E6 2008 #35**

Patrik states that a tendency of a primary insurance company's claims department to reserve claims initially to the mode of a probability distribution of claims outcomes is a contributing factor in longer claims report lags for excess contract reinsurers.

- a) Explain why a claims department might reserve claims at modal values.
- b) Explain why reserving at modal values could result in longer claim report lags for excess contract reinsurers.
- c) Construct a specific, simple numerical example to illustrate how modal reserving by the primary insurer can cause a greater IBNR need for the reinsurer than for the primary insurer.

**Solution to part a:**

- ◇ The claims department is reserving the claim at the most likely ultimate payment amount for that type of claim

**Solution to part b:**

- ◇ If the mode is lower than the treaty attachment point, reinsurers will not learn about the claim until it actually pierces the attachment point

**Solution to part c:**

- ◇ There are ten claims initially reserved at \$50 per claim. Nine of the claims develop to \$60, while one develops to \$1,000. There is a 900 XS 100 reinsurance treaty:

$$\text{Primary case reserve} = 10(50) = 500$$

$$\text{Primary IBNR} = 9(10) + 1(100 - 50) = 140$$

$$\text{Reinsurer case reserve} = 0$$

$$\text{Reinsurer IBNR} = (1000 - 100) = 900$$

**E6 2008 #36**

Given the following as of December 31, 2007:

Calendar Accident Year	Earned Premium	Adjusted Premium	Aggregate Reported Loss	Aggregate Loss Report Lag
2003	\$10,000	\$9,000	\$8,000	0.95
2004	11,000	9,000	8,000	0.88
2005	13,000	11,000	7,000	0.75
2006	15,000	13,000	6,000	0.55
2007	17,000	15,000	4,000	0.30

Calculate the IBNR as of December 31, 2007 using the Stanard-Bühlmann method.

**Solution:**

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2003	9000	0.95	8550 = 9000(0.95)
2004	9000	0.88	7920
2005	11000	0.75	8250
2006	13000	0.55	7150
2007	15000	0.30	4500

◇ Calculate the SB ELR:

$$\bullet \text{ SB ELR} = \frac{\text{reported losses}}{\text{used-up premium}} = \frac{8000+8000+7000+6000+4000}{8550+7920+8250+7150+4500} = 0.907$$

◇ Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
2003	9000	0.95	408.15 = 9000(0.907)(1 - 0.95)
2004	9000	0.88	979.56
2005	11000	0.75	2494.25
2006	13000	0.55	5305.95
2007	15000	0.30	9523.50
Total			\$18,711.41

**E6 2007 #3**

Given the following information for a reinsurance contract:

Calendar Accident Year	Earned Risk Pure Premium	On-Level Premium	Aggregate Report Lag	Chain-Ladder IBNR	Stanard- Bühlmann IBNR
2003	\$15,000	\$16,000	50%	\$5,500	\$5,845

Using Patrik's method with credibility factor 0.6, calculate the credibility IBNR estimate for 2003.



**Solution:**

$$\diamond Z = (0.60)(0.50) = 0.30$$

$$\diamond \text{Cred IBNR} = 5500(0.30) + 5845(1 - 0.30) = \boxed{\$5,741.50}$$

**E6 2007 #22**

Given the following data as of December 31, 2006:

Calendar Accident Year	Earned Risk Pure Premium	On- leveled Premium	Aggregate Reported Loss	Aggregate Report Lag
2002	\$5,000	\$7,000	\$5,000	98%
2003	7,500	8,500	6,500	87%
2004	8,000	9,500	2,000	73%
2005	9,600	10,000	7,500	65%
2006	11,500	11,500	6,500	40%
Total	41,600	46,500	27,500	

- Calculate the total IBNR as of December 31, 2006 using the chain-ladder method.
- Calculate the total IBNR as of December 31, 2006 using the Stanard-Bühlmann method.
- State and explain one disadvantage of the chain-ladder method.
- State and explain one disadvantage of the Stanard-Bühlmann method.

**Solution to part a:**

$$\diamond \text{IBNR} = \frac{5000}{0.98} + \frac{6500}{0.87} + \frac{2000}{0.73} + \frac{7500}{0.65} + \frac{6500}{0.40} - (5000 + 6500 + 2000 + 7500 + 6500) = \boxed{\$15,601.49}$$

**Solution to part b:**

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2002	7000	0.98	6860 = 7000(0.98)
2003	8500	0.87	7395
2004	9500	0.73	6935
2005	10000	0.65	6500
2006	11500	0.40	4600

◇ Calculate the SB ELR:

$$\bullet \text{SB ELR} = \frac{\text{reported losses}}{\text{used-up premium}} = \frac{5000+6500+2000+7500+6500}{6860+7395+6935+6500+4600} = 0.852$$

◇ Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
2002	7000	0.98	119.28 = 7000(0.852)(1 - 0.98)
2003	8500	0.87	941.46
2004	9500	0.73	2185.38
2005	10000	0.65	2982
2006	11500	0.40	5878.80
Total			$\boxed{\$12,106.92}$

**Solution to part c:**

◇ The chain-ladder method relies heavily on actual loss experience. Thus, if major changes in the claims department have occurred in the most recent accident years, the changes will not be properly reflected in the LDFs calculated from historical data

**Solution to part d:**

◇ The SB method is sensitive to the accuracy of the on-level earned premium. In particular, on-level earned premium can be difficult to obtain for older years

**E6 2006 #12**

Given the following information:

Calendar/Accident Year	Earned Premium (in \$000)	Adjusted Premium (in \$000)	Aggregate Reported Loss (in \$000)	Aggregate Loss Report Lag
2002	6,500	10,000	6,000	95%
2003	8,000	9,000	5,000	75%
2004	9,000	11,000	5,000	50%
2005	9,500	12,000	4,000	20%

What is the Stanard-Bühlmann expected loss ratio?

- A. Less than 50%
- B. At least 50%, but less than 60%
- C. At least 60%, but less than 70%
- D. At least 70%, but less than 80%
- E. At least 80%

**Solution:**

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2001	10000	0.95	9500 = 10000(0.95)
2002	9000	0.75	6750
2003	11000	0.50	5500
2004	12000	0.20	2400

◇ Calculate the SB ELR:

- SB ELR =  $\frac{\text{reported losses}}{\text{used-up premium}} = \frac{6000+5000+5000+4000}{9500+6750+5500+2400} = 0.828$

- Thus, the answer is E. At least 80%

**E6 2005 #38**

You are given the following information:

Calendar/Accident Year	Earned Risk Pure Premium	Rate Level		Aggregate Loss Report Lag
		Adjusted Premium	Reported Loss	
2001	\$6,000	\$8,000	\$5,000	87.5%
2002	7,500	8,000	4,000	75.0%
2003	9,000	10,000	4,000	50.0%
2004	10,000	10,000	2,000	30.0%

What is the expected loss ratio for the Stanard-Bühlmann (Cape Cod) loss reserving method?

**Solution:**

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
2001	8000	0.875	7000 = 8000(0.875)
2002	8000	0.750	6000
2003	10000	0.500	5000
2004	10000	0.300	3000

◇ Calculate the SB ELR:

- SB ELR =  $\frac{\text{reported losses}}{\text{used-up premium}} = \frac{5000+4000+4000+2000}{7000+6000+5000+3000} = \boxed{0.714}$

**E6 2004 #49**

You are given the following information:

Accident Year	Actual Earned Premium	Rate Level Adjusted Premium	Aggregate Reported Losses	Loss Development
				Factor to Ultimate
1998	\$4,500	\$6,200	\$3,200	1.000
1999	5,000	6,500	3,400	1.053
2000	5,500	7,500	3,500	1.176
2001	6,000	7,800	2,800	1.429
2002	6,500	7,800	2,100	2.000
2003	7,000	7,000	1,600	4.000
Total	\$34,500	\$42,800	\$16,600	

- Calculate the chain-ladder IBNR estimate and estimated ultimate loss ratio for accident year 2002. Show all work.
- Calculate the Stanard-Bühlmann IBNR estimate and estimated ultimate loss ratio for accident year 2002. Show all work.
- According to Patrik, what is the key innovation of the Stanard-Bühlmann method over the Bornhuetter/Ferguson method?
- According to Patrik, what is one disadvantage of the Stanard-Bühlmann method?



**Solution to part a:**

$$\diamond \text{ CL IBNR} = 2100(2 - 1) = \boxed{\$2,100}$$

$$\diamond \text{ CL ultimate loss ratio} = \frac{2100+2100}{6500} = \boxed{0.646}$$

**Solution to part b:**

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
1998	6200	1.000	6200
1999	6500	$0.950 = \frac{1}{1.053}$	$6175 = 6500(0.95)$
2000	7500	0.850	6375
2001	7800	0.700	5460
2002	7800	0.500	3900
2003	7000	0.25	1750

◇ Calculate the SB ELR:

$$\bullet \text{ SB ELR} = \frac{\text{reported losses}}{\text{used-up premium}} = \frac{3200+3400+3500+2800+2100+1600}{6200+6175+6375+5460+3900+1750} = 0.556$$

◇ Calculate the SB IBNR for AY 2002:

$$\bullet \text{ SB IBNR} = 7800(0.556)(1 - 0.50) = \boxed{\$2,168.40}$$

$$\bullet \text{ SB ultimate loss ratio} = \frac{2100+2168.40}{6500} = \boxed{0.657}$$

**Solution to part c:**

◇ The expected loss ratio is estimated from the overall reported claims experience, instead of being selected judgmentally, as in the BF method

**Solution to part d:**

◇ The SB method is sensitive to the accuracy of the on-level earned premium

**E6 2003 #44**

You are given the following information:

Accident Year	Earned	Adjusted Premium	Aggregate	Aggregate Loss Report Lag
	Risk Pure Premium		Reported Loss	
1998	\$6,500	\$8,500	\$8,500	90%
1999	7,500	9,000	6,500	80%
2000	8,700	9,500	4,500	65%
2001	9,200	9,600	2,300	40%
2002	12,100	12,100	1,500	25%

Using the Stanard-Bühlmann method, calculate the total IBNR reserve. Show all work.

**Solution:**

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
1998	8500	0.90	7650 = 8500(0.90)
1999	9000	0.80	7200
2000	9500	0.65	6175
2001	9600	0.40	3840
2002	12100	0.25	3025

◇ Calculate the SB ELR:

$$\bullet \text{ SB ELR} = \frac{\text{reported losses}}{\text{used-up premium}} = \frac{8500+6500+4500+2300+1500}{7650+7200+6175+3840+3025} = 0.835$$

◇ Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
1998	8500	0.90	709.750 = 8500(0.835)(1 - 0.90)
1999	9000	0.80	1503.000
2000	9500	0.65	2776.375
2001	9600	0.40	4809.600
2002	12100	0.25	7577.625
Total			\$17,376.35

**E6 2001 #53**

You are given the following information on ceded reinsurance as of December 31, 2000:

Accident Year	Ceded	Ceded	Ceded Incurred
	Unadjusted Earned Premium	Adjusted Earned Premium	Losses at December 31, 2000
1998	\$23,438	\$25,781	\$16,500
1999	25,568	28,125	9,000
2000	27,699	30,469	3,900

Age (months)	Percent Reported
12	16%
24	40%
36	80%
48	100%

Using the methodology outlined by Patrik, “Reinsurance,” calculate the ceded IBNR as of December 31, 2000 using the Stanard-Bühlmann technique. Show all work.

**Solution:**

◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
1998	25781	0.80	$20624.80 = 25781(0.80)$
1999	28125	0.40	11250
2000	30469	0.16	4875.04

◇ Calculate the SB ELR:

$$\bullet \text{ SB ELR} = \frac{\text{reported losses}}{\text{used-up premium}} = \frac{16500+9000+3900}{20624.80+11250+4875.04} = 0.80$$

◇ Calculate the SB IBNR:

AY	Adjusted Premium	Aggregate Loss Report Lag	SB IBNR
1998	25781	0.80	$4124.96 = 25781(0.80)(1 - 0.80)$
1999	28125	0.40	13500
2000	30469	0.16	20475.168
Total			\$38,100.13

**E6 2001 #54**

According to Patrik, “Reinsurance,” list one advantage and one disadvantage for each of the following reserve techniques:

- a) Standard American chain-ladder
- b) Bornhuetter/Ferguson

**Solution to part a:**

- ◇ Advantage – relates reserve to reported losses to date
- ◇ Disadvantage – reserve estimate is highly sensitive to reported losses to date for immature years

**Solution to part b:**

- ◇ Advantage – relates reserve to exposure
- ◇ Disadvantage – expected loss ratio is selected arbitrarily

**E6 2000 #68**

You are given the following information:

Accident Year	Adjusted	Paid Loss	Case Reserves	Aggregate
	Earned Premium			Reported Loss Lag
1996	\$200,000	\$120,000	\$27,000	90%
1997	300,000	80,000	90,000	75%
1998	350,000	45,000	135,000	50%
1999	425,000	20,000	140,000	35%

- Based on the methodology described by Patrik, “Reinsurance,” calculate an estimate of the expected loss ratio using the Stanard-Bühlmann method. Show all work.
- According to Patrik, what is the key innovation of the Stanard-Bühlmann method when compared to the Bornhuetter/Ferguson or chain-ladder methods?
- According to Patrik, what is the major problem with the Stanard-Bühlmann method?



**Solution to part a:**

- ◇ Calculate the used-up premium:

AY	Adjusted Premium	Aggregate Loss Report Lag	Used-Up Premium
1996	200	0.90	180 = 200(0.9)
1997	300	0.75	225
1998	350	0.50	175
1999	425	0.35	148.75

- ◇ Calculate the SB ELR:

$$\bullet \text{ SB ELR} = \frac{\text{reported losses}}{\text{used-up premium}} = \frac{120+27+80+90+45+135+20+140}{180+225+175+148.75} = \boxed{0.9015}$$

**Solution to part b:**

- ◇ The expected loss ratio is estimated from the overall reported claims experience, instead of being selected judgmentally, as in the BF method

**Solution to part c:**

- ◇ The SB method is sensitive to the accuracy of the on-level earned premium. In particular, on-level earned premium can be difficult to obtain for older years