Feldblum Financial

Outline

The objective of this paper is to demonstrate how an internal rate of return (IRR) model can be used to price insurance policies.

I. Introduction

Traditionally, insurance premiums included fixed underwriting profit margins (such as 5%) without theoretical justification. There are **three reasons for seeking more accurate pricing models**:

(1) **Time value of money** – Insurance cash flows occur at different times. For example, premiums/expenses are often collected/paid at policy inception, while losses are settled at a later date. Pricing models should seek to consider both the timing and magnitude of the cash flows

(2) **Competition and expected returns** – The price of a product depends on the degree of competition in the industry

(3) **Rate Base** – The traditional underwriting profit margins is a return on sales. A more appropriate rate base is a return on equity

Point of View

Insurance transactions can be examined from **two points of view**:

(1) Insurer ↔ Policyholder

- ◇ Policyholder pays premium for an insurance contract, which requires the insurer to indemnify the policyholder for covered claims
- ◇ Transactions occur in the product market
- ◇ Prices (i.e. premiums) are influenced by the supply of insurance coverage and demand for insurance services
- ◇ Profits are related to premiums and losses with no consideration of the insurer's capital structure
- ◇ Traditional rate-making procedures used this point of view

(2) Equity Provider ↔ Insurer

- ◇ Shareholders (i.e. equity providers) invest funds in an insurance company, expecting a return on that investment (i.e. capital accumulation or dividends) from the insurer
- ◇ Transactions occur in the financial market
- ◇ Expected returns driven by the risks of insurance operations
- ◇ Profits are related to assets or equity with consideration of insurance cash flows (premiums, losses and expense) ONLY to the extent that they affect transactions between the insurer and its stockholder
- ◇ IRR model uses this point of view

How are these **points of view related**?

- ◇ The supply of insurance services in the product market depends on the costs that insurers pay to obtain capital, as well as the returns achievable by investors on other uses of that capital
- ◇ The expected returns in the financial market, which are influenced by the risk of insurance operations, depend on consumers' demand for insurance services

A Non-Insurance Illustration of the IRR Model

The decision rule of the IRR model is as follows: **Accept an investment opportunity which offers a rate of return in excess of the opportunity cost of capital**. In other words, if the IRR is greater than the cost of capital, then the project is expected to be profitable and should be pursued.

As a simple **example**, consider a firm with \$250,000 of annual revenue and \$50,000 of annual expenses. For \$100,000, the firm can purchase new equipment with a two year life span that would increase annual revenue to \$300,000 and decrease annual expenses to \$35,000. Should the firm purchase the new equipment?

Before we can answer this, we need assumptions around timing and taxes:

- ◇ Purchase costs are incurred at the beginning of the year
- ◇ Increases in revenue and decreases in expenses occur at the end of each year
- ◇ No federal income taxes

We need to determine the IRR and compare it to the cost of capital. The IRR is the rate of return needed to set the net present value of cash flows to zero. In this case, we want to solve for R in the following equation:

$$0 = -100000 + \frac{65000}{1+R} + \frac{65000}{(1+R)^2}$$

where 65,000 = (300,000 - 250,000) + (50,000 - 35,000). Using the IRR function within the CBT environment, we find that $R = 19.4\%$. Assuming the cost of capital is less than 19.4% per annum, then the firm should purchase the equipment.

Quick Aside on the CBT IRR Function

Similar to Excel, the CBT environment has an IRR function. Here are the steps needed to solve this problem using the function:

(1) We are going to use two columns. To make it easy on the graders, let's label the columns "Time" and "Cash Flow"

(2) Under "Time," enter 0, 1 and 2 since the cash flows take place at times 0, 1 and 2. Note that the times are NOT inputs into the IRR function. Instead, the IRR function assumes that the cash flows provided occurs at times 0, 1, 2, etc. However, we should label the time periods for clarity

(3) Under "Cash Flow," let's enter -100,000 in the cell next to Time 0, 65,000 in the cell next to Time 1 and 65,000 in the cell next to Time 2

(4) In a separate cell, enter the following IRR function: "=IRR(...)", where the "..." references the cash flows. Make sure to label this cell as IRR for clarity and ease of grading

The CBT environment should show 19.4%.

Insurance IRR Models

In the non-insurance example, there was an initial outflow (the purchase of the equipment) followed by future revenues (the $65,000 from additional revenue and reduced expenses). Property/liability insurance operations **appear to show the opposite pattern**: the insurer collects premiums before paying losses. However, this **ignores the equity commitments that support the insurance operations**.

Two aspects of insurance operations that reflect the equity-holders' perspective are incorporated in IRR pricing models:

(1) When an insurer writes a policy, part of the premium is used to pay acquisition, underwriting and administrative expenses. The remaining premium dollars are invested to support the unearned premium reserve and the loss reserve

(2) Insurance companies commit surplus to ensure that that company has sufficient capital to withstand unexpected losses

The commitment of surplus underlies the equity flows of an insurer's owners. The owners must provide funds to allow the firms to write the policy. These funds are **cash outflows**. The return on these funds occurs in future years as the policy expires, losses are paid and the surplus is freed. These returns are cash inflows. Now we can see the similarities between the non-insurance IRR models and the insurance IRR models.

Equity Flows

Equity flows represent the cash outflows and cash inflows for the equityholders. These cash flows are impacted by insurance transactions (premiums, losses, expenses, etc.) in that higher levels of underwriting and investment income reduce the amount of surplus commitment needed from equityholders. *At this point, Feldblum goes into some detail on the level of surplus associated with certain lines of business. Since this is explored further later on in the paper, I will skip it for now and come back to it later.*

An Equity Flow Illustration

The following example is a modified version of the one shown in the paper. Suppose an insurer has an opportunity to write a new program with the following characteristics:

- ◇ Premium of $1,000 collected and earned at time 0
- ◇ Expected losses of $500 and $200 paid at time 1 and time 2, respectively
- ◇ Expenses are 30% of premium and are paid at time 0
- ◇ Loss reserves are established at time 0 and there are no unearned premium reserves
- ◇ Before-tax investment yield is 10%
- ◇ Tax rate on investment income is 35%
- ◇ Cost of capital is 8%
- ◇ The required reserve to surplus ratio 2:1

To determine the IRR for this model, it's simplest to create a table. I use the following table for every IRR problem:

	Time 0	Time 1	Time 2
Premium	1,000	0	0
Expense	300	0	0
Paid Loss	0	500	200
Loss Reserve	700	200	0
Required Surplus	350	100	0
Required Assets	1,050	300	0
Beginning Assets	1,000	1,118.25	319.50
Payments	300	500	200
Equity Flow (from Insurer's Perspective)	350	–318.25	–119.50
Equity Flow (from Equityholders' Perspective)	−350	318.25	119.50
Ending Assets	1,050	300	0

Here are the general formulas I used for the example above:

- Required Surplus = $\frac{\text{Loss Reserve}}{\text{Reserve to Surplus Ratio}}$
- Required Assets = Loss Reserves + Required Surplus
- Beginning Assets = Ending Assets × After-Tax Investment Yield (with Beginning Assets at Time 0 equal to Premium)
- Ending Assets = Required Assets
- Equity Flow (from Insurer's Perspective) = Required Assets + Payments – Beginning Assets
- Equity Flow (from Equityholders' Perspective) = –Equity Flow (from Insurer's Perspective)

To illustrate the formulas, let's look at time periods 0 and 1:

- Time 0
  - Expense = 1,000(0.30) = 300
  - Payments = 300
  - Required Surplus = $\frac{700}{2}$ = 350
  - Beginning Assets = Premium = 1,000
  - Required Assets = Ending Assets = 700 + 350 = 1,050
  - Equity Flow (from Insurer's Perspective) = 1,050 + 300 − 1,000 = 350
  - Equity Flow (from Equityholders' Perspective) = –350
- Time 1
  - Payments = 500

- Loss Reserve = 700 - 500 = 200
- Required Surplus = $\frac{200}{2} = 100$
- Required Assets = Ending Assets = 200 + 100 = 300
- Beginning Assets = $1,050[1 + 0.10(1 - 0.35)] = 1,118.25$
- Equity Flow (from Insurer's Perspective) = 300 + 500 − 1,118.25 = –318.25
- Equity Flow (from Equityholders' Perspective) = 318.25

We want to solve for R in the following equation: $0 = -350 + \frac{318.25}{1+R} + \frac{119.50}{(1+R)^2}$. Notice that we are using the equity flows from the **equityholders' perspective**. Using the IRR function within the CBT environment, we find that $R = 19.5\%$. Since R is greater than the cost of capital ($19.5\% > 8\%$), the premiums are adequate from the equityholders' perspective and the program should be pursued

There is one thing that should be noted in the example above:

◇ Generally speaking, the UEPR is often assumed to be zero for these problems. The ultimate losses are assumed to be known at Time 0 and the initial surplus commitment is obtained by applying a reserve to surplus ratio to the loss reserves. However, it's possible for an exam problem to state the initial surplus requirement as a function of the UEPR. Make sure to look for this when working exam problems

II. Surplus

A insurance contract is a promise to compensate policyholders for future claims. An insurer must have financial assets to fulfill this promise. Unlike the fixed assets held by a manufacturer (ex. plants/equipment) which can be objectively measured, an insurer's surplus and its allocation to lines of business and time periods is theoretical. Thus, an IRR model must assume a relationship between surplus commitment and insurance transactions.

The Individual Firm and the Industry

The difference between insurance pricing and the uses of IRR models in other industries has several implications:

(1) In other industries, capital investments reflect a deliberate strategy. If a firm anticipates larger returns, it should invest more funds. In the insurance industry, capital markets are assumed to be efficient which means that the industry is neither over- nor under-capitalized. However, an individual insurer may be over- or under-capitalized due to the favorable or adverse operating results in the past. This implies that the actual surplus held by an insurer is not necessarily a deliberate strategy

(2) The IRR model focuses on the **capital/financial market** (return on equity vs. cost of capital) whereas the prices of insurance contracts are determined in the **product market** (supply and demand of insurance policies)

- ◇ At the industry level, the capital and product markets are connected. If market prices are inadequate and industry returns are below the cost of capital, then firms will leave the industry and prices will rise
- ◇ At the firm level, this relationship doesn't necessarily hold. An individual firm's IRR may be depressed by operating inefficiencies even when industry rates are adequate

Surplus Allocation

As mentioned earlier, the allocation of an insurer's surplus is theoretical. Surplus exists for the company as a whole and is available to all lines of business. However, an allocation of surplus to line of business is needed for the IRR model. There are two important questions related to this allocation:

(1) How is surplus allocated?

- ◇ Base – Actuaries generally allocate surplus in proportion to a base, such as written premium, statutory reserves or the present value of future loss payments
- ◇ Timing – Actuaries must understand when the surplus is committed and when the surplus is freed

(2) Should the surplus allocation depend on the type of policy?

- ◇ For example, does a retrospectively rated Workers' Compensation policy require less surplus than a guaranteed cost policy?

Premiums and Reserves

If we use **premiums** as our surplus allocation base, it implies the following:

- ◇ Required surplus varies directly with premium. If one line has twice the premium of a second line, then it receives twice the surplus commitment
- ◇ Surplus is committed when the premium is written, and it is released when the policy expires

If we use **reserves** as our surplus allocation base, it implies the following:

- ◇ Required surplus varies directly with reserves. If one line has twice the reserves of a second line, then it receives twice the surplus commitment
- ◇ If the allocation base is loss and expense reserves only, then surplus is committed when the losses occur, and it is released when losses are paid

- ◇ If the allocation base includes the loss portion of the unearned premium reserves in addition to the loss reserves, then surplus is committed when the policy is written, and it is released when the losses are paid

When more surplus is allocated to a line of business, the internal rate of return moves closer to the company's after-tax investment yield. This is because an increase in surplus decreases the internal rate of return. Generally speaking, the insurance industry's cost of capital exceeds the after-tax investment yield so any decrease in the internal rate return will move it closer to the investment yield.

Long- and Short-Tailed Lines

Slow paying lines with large loss reserves (ex. Workers' Compensation, General Liability) receive a larger allocation of surplus if reserves are used as the base instead of premium. This is due to the period of time in which surplus is committed.

As an **example**, suppose an insurer with $25 million in surplus writes two lines of business with the following characteristics over the next year:

	Line A	Line B
Written Premium	$20 million	$20 million
Loss Ratio	50%	40%
Average Time From Loss to Payment	3 years	1 years

Allocation of surplus by **premiums** gives the following:

- ◇ Line A Allocated Surplus = $\frac{20}{20+20}(25) = \$12.5$ million
- ◇ Line B Allocated Surplus = $\frac{20}{20+20}(25) = \$12.5$ million

Allocation of surplus by **reserves** gives the following:

- ◇ Line A Steady State Reserves = $20(0.50)(3) = \$30$ million
- ◇ Line B Steady State Reserves = $20(0.40)(1) = \$8$ million
- ◇ Line A Allocated Surplus = $\frac{30}{30+8}(25) = \$19.7$ million
- ◇ Line B Allocated Surplus = $\frac{8}{30+8}(25) = \$5.3$ million

Note the term "steady state reserves" above. By "steady state," we are referring to the reserves at any point in time in a steady state environment. For Line A, there are $20(0.50) =$ *$10 million in losses each year. Since the average time loss to payment is 3 years, there are $30 million in reserves at any point in time.*

If we had the underwriting income for each line of business over the next year, we could calculate the return on equity under each allocation basis above as $\text{ROE} = \frac{\text{UW Income}}{\text{Allocated Surplus}}$.

Insurance Risks

Surplus protects the insurer against the following risks:

- ◇ **Asset risk** – Risk that financial assets will depreciate (ex. bonds will default or stock prices will drop)
- ◇ **Pricing risk** – Risk that incurred losses and expenses will be greater than expected at policy expiration
- ◇ **Reserving risk** – Risk that loss reserves will not cover ultimate loss payments
- ◇ **Asset-liability Mismatch risk** – Risk that changes in interest rates will affect the market value of certain assets (ex. bonds) differently than that of liabilities
- ◇ **Catastrophe risk** – Risk that unforeseen losses (ex. hurricanes, earthquakes) will depress the return realized by the insurer
- ◇ **Reinsurance risk** – Risk that reinsurance recoverables will not be collected
- ◇ **Credit risk** – Risk that agents will not remit premium balances or that insureds will not remit accrued retrospective premiums

Some of the risks above (ex. pricing risk, catastrophe risk) occur during the policy period. The other risks continue until all losses are paid. For this reason, surplus is often allocated based on unpaid losses or a combination of unpaid losses and written premiums.

Policy Form

Risk varies by policy form:

- ◇ Occurrence contracts vs. Claims-made contracts
  - Occurrence contracts cover claims that occur during the policy term
  - Claims-made contracts cover claims that are reported during the policy term
  - Since they only cover reported claims, claims-made contracts eliminate much of the Incurred But Not Reported (IBNR) liability, which reduces the loss uncertainty. Thus, less surplus is needed to support these contracts
- ◇ Service contracts
  - Under service contracts, the insurer handles claims but does not incur loss liabilities. Thus, no insurance risk exists and no surplus is required for statutory financial statements

◇ Retrospective rating contracts

- Under retrospective rating, losses are fully reimbursed by the policyholder in the **primary layer**. There is "credit risk" present here due to the possibility that the insured fails to pay the premiums
- Outside of the primary layer (i.e. losses hit the loss limit or premiums hit the maximum premium), significant insurance risk to the insurer exists as they won't receive additional premium. This places the risk for these contracts between that of a service contract and a prospectively rated contract (such as occurrence and claims-made contracts)

◇ Other policy form examples include excess coverage and large deductible policies

As shown above, risk varies by policy form. Theoretically, we should allocate more surplus to riskier contracts. If we treat all policy forms equally, we would **overstate the risk** on retrospectively rated policies and **understate the risk on excess coverage**. If we only consider premiums and reserves, we would **understate the risk** on retrospectively rated, large deductible and excess policies. This is due to the fact that these three policy types cover claims that occur in higher loss layers where loss fluctuations are greater.

III. Potential Pitfalls in IRR Analyses

There are two common capital budgeting techniques that consider the time value of money:

(1) IRR – determines the interest rate that sets the present value of cash inflows to the present value of cash outflows. If this interest rate exceeds the cost of capital, then the project should be pursued. Otherwise, the project should be rejected. This is the technique we have been describing throughout the paper

(2) Net Present Value (NPV) – uses the **cost of capital** to discount all cash flows to the same time. If the sum of the discounted values is positive, then the project should be pursued. Otherwise, the project should be rejected.

As an **example** of the NPV technique, let's consider the equity flow illustration for an insurer that we looked at earlier. In that example, the equity flows **for the equityholder** were −350, 318.25 and 119.50 for times 0, 1 and 2, respectively. Rather than solving for the IRR, we must discount these flows to time 0 using the cost of capital of 8%. Thus, we find the following:

$$NPV = -350 + \frac{318.25}{1.08} + \frac{119.50}{(1.08)^2} = 47.13$$

Since the NPV is positive, we should pursue the program. In this case, the IRR and NPV techniques produced the same "accept or reject" decision. In fact, these two methods typically provide the same "accept or reject" decision. However, they may give different answers under the following scenarios:

- ◇ Budget constraints
- ◇ Mutual exclusive projects
- ◇ Unusual cash flows

A **criticism** of an IRR analysis is that it doesn't optimize the net worth of the corporation. Some argue that the NPV technique does this and should be used instead. Others counter this by stating that these arguments are not material from a practical standpoint.

Cash Flow Patterns

The cash flow patterns affect the number of positive solutions to the internal rate of return equation. The normal cash flow pattern is an initial cash outflow followed by a series of inflows. In this case, there is a single sign reversal as time progresses and one positive real root to the IRR equation. However, it's possible for projects to display an initial outflow followed by an inflow followed by another outflow. In this case, there are two sign reversals and "at most" two positive real roots to the IRR equation. If two positive real roots are produced, zero, one or both of them might be reasonable. To resolve this issue, analysts should seek to reframe the equation so that only one sign reversal (and hence, only one answer) occurs.

Oversimplifications

In general, multiple sign reversals in projected cash flows result from inaccuracies or oversimplifications. For example, suppose a specific cash flow item is spread evenly across four quarters as a simplification. If this is not realistic, this simplification may cause a sign reversal when combined with other cash flows that are allocated to each quarter more precisely.

Although sign reversals are often caused by oversimplifications, they can be legitimate as well. For example, an unexpected IBNR loss may cause a cash outflow from investors in the middle of the stream of inflows.

Mutually Exclusive Projects and Reinvestment Rates

When projects are mutually exclusive (ex. aggregate budget constraints or projects accomplish the same goal), NPV and IRR techniques may not give the same ranking of projects.

An an **example**, suppose an insurer is comparing the NPVs of two projects at various interest rates:

Project	Cash Flow at Time 0	Cash Flow at Time 1	Cash Flow at Time 2
A	–12,000	10,000	6,500
B	–12,000	5,000	12,500

Project	NPV at 10%	NPV at 15%	NPV at 20%	NPV at 25%	NPV at 30%
A	2,463	1,611	847	160	–462
B	2,876	1,800	847	0	–757

The NPV analysis shows that Project B is preferable to Project A at interest rates below 20%, whereas Project A is preferable to Project B at interest rates above 20%. Summarized another way, it is often wiser to defer income for a larger total return when interest rates are low (such as Project B). On the flip side, it is wiser to achieve income early when interest rates are high (such as Project A).

The IRR for Project A is 26.3% and the IRR for Project B is 25%. Based on the IRR rules, we should pursue Project A. This line of thinking leads to **another criticism** of the IRR analysis. Project A is only preferable **if the revenue received in the first year ($10,000) can be invested at 26.3%**. In reality, the firm's cost of capital is 15%, which represents the return that the firm's owners can receive on their funds. The IRR analysis **incorrectly mixes the interest rate that equates real values of inflows and outflows with the interest rate at which the firm can invest the funds it receives**.

Feldblum states that this point is not valid for the IRR insurance pricing model for the following reasons:

(1) The IRR pricing model is used to set statewide manual rates, not to price individual policies. If the cost of capital is 15%, but the pricing model shows an IRR of 20%, the insurer can reinvest the revenue it receives by writing more policies. Assuming the insurer can grow at the internal rate of return and maintain the same quality of risks, the IRR assumptions are correct

(2) When using the IRR pricing model to determine the UW profit provision, the actuary selects a premium rate that equalizes the IRR and the cost capital. This eliminates the difference between NPV and IRR analyses

Practical Criticisms

As discussed, an IRR analysis rejects a project if the IRR is less than the cost of capital (even if the IRR is positive and greater than the investment yield). This might confuse a regulator as a positive IRR appears to indicate profitability. Furthermore, an IRR above the investment yield seems to indicate that rates are more than sufficient.

From an intuition standpoint, the NPV method wins here. Since the NPV discounts cash flows at the cost of capital, it would show a negative number in the case where the IRR is below the cost of capital. A negative NPV clearly demonstrates that rates are not adequate. However, in an IRR pricing model, **as costs increase, the internal rate of return drops slowly**. This slow drop can create positive values even as premiums become severely inadequate.

This occurs because the required surplus in an IRR pricing model changes as costs change. For example, suppose required surplus is determined by a reserve to surplus ratio. Then, as costs increase, the surplus and investment income will also increase. Although the total return is inadequate, the added investment income offsets some of the UW loss, and the IRR declines more slowly with increases in loss.

This differs from a utility company where the equity base against which return are measured is a fixed amount (i.e. doesn't change as costs change).

Premium Inadequacies and IRR Analyses

In the case of rising costs but depressed premiums, the implied equity flow assumptions are not reasonable. In reality, equityholders will provide less capital or none at all. If properly interpreted, an IRR model will show this.

As an **example**, suppose the following is true for a policy:

- ◇ Premium of $10,000 is collected at policy inception
- ◇ One loss will be paid four years after policy inception
- ◇ The insurer funds the loss with a four year zero-coupon bond yielding 10% per year
- ◇ Required surplus assumes a 2:1 ratio of reserves to surplus
- ◇ Cost of capital is 15%

At an **expected loss of $12,000**, we obtain the following:

- ◇ At policy inception, total assets after equity contributions are $18,000. This is calculated as follows:
  - At policy inception, the UW loss is 10,000 - 12,000 = -2,000. Equityholders will fund the loss
  - Equityholders will also provide 6,000 of supporting surplus, where 6,000 = 12,000/2
  - Thus, at policy inception, the equity contributions are equal to 8,000 and total assets are equal to 10,000 + 8,000 = 18,000
- ◇ The total assets grow to $26,354 after four years in a zero-coupon bond
- ◇ At time 4, the loss of $12,000 is paid and $14,354 is return to the equityholders
- ◇ The IRR found by solving the following equation: $0 = -8000 + \frac{14354}{(1+R)^4}$. Thus, R = IRR = 15.74%. Since this is greater than the cost of capital of 15%, this policy is acceptable

At an **expected loss of $15,000**, we obtain the following:

- ◇ Before considering the IRR analysis, we can see that this contract is unprofitable. The premium of $10,000 accumulates to $14,641 after four years at 10%. This means that the premium plus investment income is not enough to cover the loss of $15,000. Part of the loss must be funded with existing surplus
- ◇ An IRR analysis will lead to the same conclusion if interpreted properly
- ◇ At policy inception, total assets after equity contributions are $22,500 (10,000 + 12,500). These grow to $32,942 after four years in a zero-coupon bond
- ◇ At time 4, The loss of $15,000 is paid and $17,942 is return to the equityholders
- ◇ The IRR found by solving the following equation: $0 = -12500 + \frac{17942}{(1+R)^4}$. Thus, R = IRR = 9.46%. Since this is less than the cost of capital of 15%, this policy is not acceptable. Furthermore, since the IRR is less than the investment yield, the operating ratio exceeds 100%, and existing surplus must be used to fund the loss

We must increase the expected losses considerably before a negative IRR occurs. But it's clear that we have profitability issues well before that.

At the end of the day, the problem boils down to rate filing presentation and acceptance by regulators. When the contract is unprofitable, Feldblum suggests that the actuary show the negative expected NPV (from the NPV technique) and the insufficiency of net premiums plus investment income to fund the losses. However, the IRR analysis can still be used internally.

